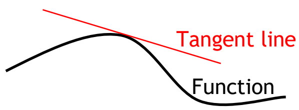
Definition of a Derivative

The derivative of a function represents the slope of the tangent line to the function.

Tangent Line



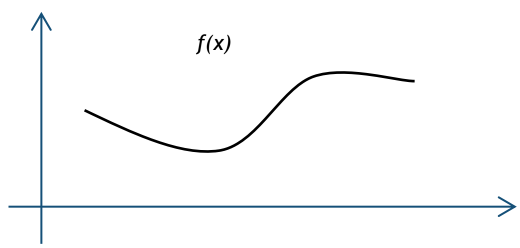
Slope of a line

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

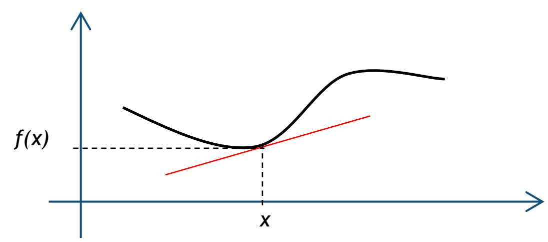
*we need 2 points on the line to get the slope

First-Principles Definition of a Derivative

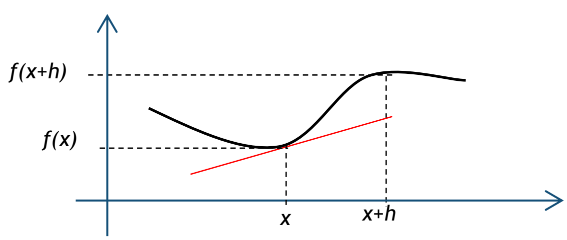
1. Let's start by defining a function $f(x)$.



2. Let's say we want to find the derivative at point x (i.e. the slope of the tangent line at point *x*).



3. To get the slope of a line we need **two** points on that line. A tangent intersects the curve at only **one** point, so we start by picking a second point on the curve, let's call it *x*+*h*.



4. Let's calculate the slope of the line between point *x*, and point *x*+*h*.

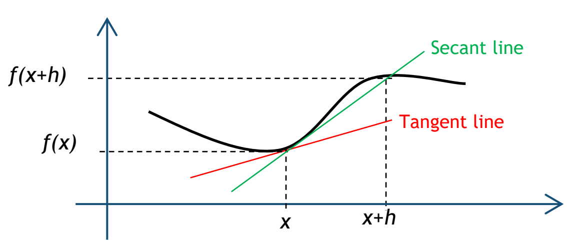
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{f(x+h) - f(x)}{(x+h) - x}$$

$$m = \frac{f(x+h) - f(x)}{\cancel{x} + h - \cancel{x}}$$

$$m = \frac{f(x+h) - f(x)}{h}$$

5. We have the slope of the secant line between point *x*, and point *x*+*h*, we want the slope of the tangent line at point *x*.



We need to reduce the gap between point *x*, and point *x*+*h*.

6. To do this we will take *h* to be smaller and smaller until it is almost zero. In other words we want to know the limit of the slope function as *h* approaches zero.

$$\mathrm{m} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

This represents the slope of the tangent line at point *x*, i.e. the derivative at point *x*.

$$\mathrm{f}'(\mathrm{x}) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

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Email : learning@brocku.ca
Brocku.ca/learning-services
905-688-5550 ext. 5774