## Calculating Basic Limits

A limit is the value a function would have if the function existed at the desired input value. We say that the function approaches some value as the input approaches some value.

## Basic Limits

Sometimes we have to do a little algebra to calculate the limit. There are two types of calculations you will likely see in first-year calculus.

| Rational <br> has factoring |
| :--- |
| These types of calculations occur when the limit |
| we are taking results in the denominator of the |
| function becoming zero. |
| For Example: |
| $\lim _{x \rightarrow-2} \frac{x^{2}+x-2}{x+2}$ |

If we input $x=-2$ into the equation, we end up with 0 in the denominator.

Since we cannot divide by zero we will first have to factor the numerator.

$$
=\lim _{x \rightarrow-2} \frac{(x+2)(x-1)}{x+2}
$$

Now, we can cross out the $(x+2)$ factors in the numerator and denominator.

$$
\begin{gathered}
=\lim _{x \rightarrow-2} \frac{(x+2)(x-1)}{x+2} \\
=\lim _{x \rightarrow-2} x-1
\end{gathered}
$$

We've eliminated the divide by zero problem so we can now take the limit by plugging in $x=-2$.

$$
=-2-1=-3
$$

Therefore: $\quad \lim _{x \rightarrow-2} \frac{x^{2}+x-2}{x+2}=-3$

## Conjugate <br> has square roots

These types of calculations occur when the limit we are taking results in a divide by zero situation and the function has a square root.

For Example:

$$
\lim _{x \rightarrow-2} \frac{\sqrt{x+3}-1}{x+2}
$$

If we input $x=-2$ into the equation, we end up with 0 in the denominator.

Since we cannot divide by zero and we cannot factor we will have to multiply by the conjugate of the binomial containing the square root.

A conjugate is when we change the sign between two terms. $(a+b)$ and $(a-b)$ are conjugates.

$$
\begin{array}{l|l}
\lim _{x \rightarrow-2} \frac{\sqrt{x+3}-1}{x+2} \times \frac{\sqrt{x+3}+1}{\sqrt{x+3}+1} & \begin{array}{l}
\text { FOIL the } \\
\text { numerator }
\end{array} \\
=\lim _{x \rightarrow-2} \frac{(\sqrt{x+3})^{2}-1}{(x+2)(\sqrt{x+3}+1)} & \begin{array}{l}
\text { Don't FOIL the } \\
\text { denominator }
\end{array} \\
=\lim _{x \rightarrow-2} \frac{x+3-1}{(x+2)(\sqrt{x+3}+1)} & \\
=\lim _{x \rightarrow-2} \frac{x+2}{(x+2)(\sqrt{x+3}+1)} & \begin{array}{l}
\text { Cancel the } \\
\text { factors }
\end{array} \\
=\lim _{x \rightarrow-2} \frac{1}{\sqrt{x+3}+1} & \\
=\frac{1}{\sqrt{-2+3}+1}=\frac{1}{2} & \text { Take the limit }
\end{array}
$$

Therefore: $\quad \lim _{x \rightarrow-2} \frac{\sqrt{x+3}-1}{x+2}=\frac{1}{2}$

