Estimation of a Nonlinear Taylor Rule Using Real-Time U.S. Data

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Abstract

This paper extends the work in Orphanides (2003) by re-examining the empirical evidence for a Taylor rule in a nonlinear framework. In doing so, it updates the Greenbook dataset used by the afore mentioned author to the most recent available period.

A three-regime threshold regression model is utilized to capture the possibly asymmetric policy reaction function used by the U.S. Federal Reserve. The theoretical foundations for such an approach to monetary policy are discussed in Orphanides and Wilcox (2002).

Our results indicate that the estimated Taylor rule for the U.S., based on real-time Greenbook data for the period 1982:3-2003:4, is probably nonlinear.

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1 Introduction

John Taylor (Taylor, 1993) - formalized the notion that monetary policy in the United States could be usefully described by a simple rule, according to which, the Federal Reserve sets the target for the Federal Funds Rate in order to establish the equilibrium level of the real interest rate; with adjustments aimed at correcting deviations of the rate of inflation from its target, and of output from potential. Much of the empirical literature on the Taylor rule has focused on a forward-looking specification in a departure from the original backward-looking rule - see Clarida et al. (1998, 2000), Orphanides (2003) among others. A forward-looking policy reaction function is arguably a better characterization of the objectives of central banks.

A number of recent policy evaluation studies highlight the importance of using information that was actually available to the policy makers when they were making their interest-setting decisions - Orphanides (2001, 2002, and 2003). These studies show that estimates of policy reaction functions based on retrospective (*ex-post*) data often lead to serious mischaracterizations of the intended policy by the monetary authority. Orphanides (2002) argues that the Great Inflation of the 1970's, which is generally viewed as the most dramatic failure of macroeconomic policy in the United States since the founding of the Federal Reserve, does not necessarily reflect the absence of a coherent and disciplined monetary policy during this period. It is shown that - even if the Fed had been following a standard forward-looking Taylor rule - severe underestimates of future inflation and of the natural rate of unemployment that occurred in real-time decision making, would have led to settings of the Federal Funds Rate substantially below the levels indicated using *ex-post* data. The Federal Funds Rate during the 1970's was very close to the level recommended by a standard forward-looking Taylor rule implemented with *ex-post* data.

Molodtsova et al. (2008) find that estimated Taylor rules based on revised (*ex-post*) data and real-time data differ more for Germany than for the U.S. Further, Taylor rules estimated with real-time data point to differences between U.S. and German monetary policies. Finally, they report evidence of out-of-sample predictability for the dollar/deutchemark nominal exchange rate using forecasts for interest rate differentials from Taylor rules estimated with real-time data. Taylor rules based on revised data do not improve nominal exchange rate predictability.

The use of real-time data simplifies estimation by allowing researchers to use nonlinear least squares as opposed to IV or GMM procedures that are required when retrospective data is being used. Further, estimation using real-time data avoids identification issues in forward-looking Taylor rules. An and Schorfheide (2007) and Cochrane (2007) argue that there are important parameter identification issues in DSGE models. Given that monetary policy rules characterize many DSGE models, identification may be a problem. Mavroeidis (2008) used identification-robust procedures (see Stock and Wright (2000) and Kleibergen (2005)) in his evaluation of the U.S. Taylor rule and found that identification issues should be taken seriously.

More recent theoretical and empirical work has explored the possibility that the policy reaction function is nonlinear. Dolado et al. (2005) and Surico (2006) argue that the Federal Reserve is minimizing an asymmetric loss function that assigns different weights to positive and negative deviations of inflation from its target and positive and negative values of the output gap. Orphanides and Wilcox (2002) search for a specification of the policymaker's loss function that would lead to an asymmetric response to disinflation. In what they describe as an "opportunistic approach" to disinflation, when inflation is moderately above or below its long run objective - within a band defined by an upper and lower threshold- the Federal Reserve should not take deliberate action by changing the Fed Funds rate. Instead, the Fed should rely on favourable aggregate supply and/or demand shocks to steer inflation toward its long run target. On the other hand, should inflation veer outside the inaction band, the Fed should adjust the Fed Funds rate aggressively.

Qin and Enders (2008), using real-time data for the U.S., compare the in-sample and the out-of-sample properties of a number of linear and nonlinear Taylor rules as a means of model selection. Taylor and Davradakis (2006) test for nonlinearity in the conduct of monetary policy by the Bank of England using *ex-post* data.

This paper extends the work in Orphanides (2003) by re-examining the empirical evidence for a Taylor rule in a nonlinear framework. In doing so, it updates the Greenbook data set used by the afore mentioned author to the most recent available period (2003:4). A three-regime threshold regression model, motivated by Orphanides and Wilcox (2002), is utilized to capture the possibly asymmetric policy reaction function of the U.S. Federal Reserve. The theoretical foundations for such an approach to monetary policy are further discussed Section 2.

The empirical model employed in this paper is very general and nests a number of more restrictive models in it. One is a two-regime model with active monetary policy above and below the single threshold, another is a model with active monetary policy when inflation is running above the single threshold, but inaction when it lies below the threshold. The most restrictive case is represented by a linear model such as the one considered in Orphanides (2003). The thresholds in the nonlinear models are estimated endogenously and testing down from the most general model to the linear case is accomplished through sequential likelihood-ratio tests. A similar modelling strategy was followed by Taylor and Davradakis (2006) who used retrospective data for the U.K.. Our results indicate that the estimated U.S. Taylor rule for the period 1982:3-2003:4, using data that were available to the policy makers in real time, is probably nonlinear.

The following Section discusses an encompassing empirical specification of the Taylor rule. Section 3 reviews the econometric methodology. Section 4 reports tests for asymmetric policy response and Section 5 concludes.

2 An Encompassing Policy Reaction Function

The original Taylor rule, henceforth referred to as the classic rule, postulates that the target for the short-run nominal interest rate, i_t^*, is set as follows:

$$i_t^* = r^* + \pi_t + \zeta_\pi(\pi_t - \pi^*) + \zeta_y y_t \tag{1}$$

where r^* is the long-run equilibrium real interest rate (assumed to be constant and known to the central bank), π^* is the target inflation rate (also assumed to be constant), π_t is the the inflation rate at time t, and y_t is the percent real output gap. Potential real output is assumed to be exogenous. Equation (1) can be written as:

$$i_t^* = r^* - \zeta_\pi \pi^* + (1 + \zeta_\pi)\pi_t + \zeta_y y_t \ . \tag{2}$$

Clarida et al. (1998) point out that $\zeta_\pi > 0$ and $\zeta_y > 0$ reflect stabilizing behaviour on the part of the central bank. For example, a one percent increase in inflation will result in an upward revision of the target for the overnight interest rate by $1 + \zeta_\pi$ - equation (2). For $\zeta_\pi > 0$, the latter exceeds the rise in inflation and raises the target real rate. This response slows down real economic activity to counter inflation. If on the other hand, ζ_π and/or ζ_y are negative, monetary policy is "accommodative". In the latter case, Clarida et al. (2000) argue that self-fulfilling bursts of inflation and output may be possible.

Orphanides (2003) shows that Friedman-type money growth rules can be reformulated along the lines of (2). Specifically,

$$i_t^* = r^* - \zeta_\pi \pi^* + (1 + \zeta_\pi)\pi_t + \zeta_{\Delta y}(\Delta y_t - \Delta y^*) \tag{3}$$

where the Fed Funds Rate targets deviations of real output growth from potential output growth.

Empirical work on Taylor-type rules has been based on two variations on equations (2) and (3). The first concerns forward looking rules, where inflation and the output gap are replaced with expected values of these variables.

Expectations are assumed to be rational conditional on information available through time $t-1$.

The second variation is based on interest rate smoothing exercised by the monetary authority as suggested by Goodfriend (1991). Interest rate smoothing may be the result of central bank concerns about financial market disruption in the face of drastic changes in the Fed Funds Rate. Similarly, interest rate smoothing may help avoid large policy reversals that would be damaging to central bank credibility. The partial adjustment hypothesis, outlined below, shows that the Fed Funds Rate adjusts to close the gap between the actual rate and its target gradually:

$$i_t - i_{t-1} = (1-\phi)\left[i_t^* - i_{t-1}\right] \tag{4}$$

where $1 > \phi > 0$, and $(1-\phi)$ represents the degree of interest rate smoothing. Equation (4) is re-arranged as:

$$i_t = \phi i_{t-1} + (1-\phi) i_t^*. \tag{5}$$

Substituting in (5) the expression for the target Fed Funds Rate resulting from equations (2) and (3), we get a dynamic expression describing the Fed Funds Rate in the short and medium run:

$$i_t = \phi i_{t-1} + (1-\phi)\left[r^* - \zeta_\pi \pi^* + (1+\zeta_\pi)\pi_t + \zeta_{\Delta y}(\Delta y_t - \Delta y^*) + \zeta_y y_t\right]. \tag{6}$$

For estimation purposes, equation (6) will be written as:

$$i_t = \theta_0 + \theta_i i_{t-1} + \theta_\pi \pi_t + \theta_{\Delta y}(\Delta y_t - \Delta y^*) + \theta_y y_t + \epsilon_t \tag{7}$$

where ϵ is an i.i.d. disturbance with zero mean and constant variance. Further, $\theta_0 = (1-\phi)(r^* - \zeta_\pi \pi^*)$, $\theta_i = \phi$, $\theta_\pi = (1-\phi)(1+\zeta_\pi)$, $\theta_{\Delta y} = (1-\phi)\zeta_{\Delta y}$, and $\theta_y = (1-\phi)\zeta_y$.

Following Orphanides (2003), we utilize the following empirical specification of (7)

$$i_t = \theta_0 + \theta_i i_{t-1} + \theta_\pi \pi_{t+3}^a + \theta_{\Delta y} \Delta^a y_{t+3} + \theta_y y_{t-1} + \epsilon_t \tag{8}$$

where $\pi_{t+3}^a = p_{t+3} - p_{t-1}$ is the year-ahead inflation forecast based on information available at time $t-1$. $\Delta^a y_{t+3} = (y_{t+3} - y_{t-1}) - (y_{t+3}^* - y_{t-1}^*)$ is the year-ahead growth forecast relative to potential. Finally, y_{t-1} is the estimate of the output gap for quarter $t-1$ available during quarter t.

Equation (8) nests a number of linear policy reaction functions. For example, setting $\theta_i = \theta_{\Delta y} = 0$ yields the inflation forecast version of the classic Taylor rule. The restriction $\theta_i = 0$, $\theta_{\Delta y} = -\theta_y > 0$ is equivalent to the classic Taylor rule that targets both inflation and output-gap forecasts.

Setting $\theta_i = 1$, $\theta_y = 0$ and $\theta_{\Delta y} > 0$ corresponds to a natural growth targeting rule. Finally, setting $\theta_\pi = \theta_{\Delta y} = \theta_y = 0$, and $\theta_i = 1$ corresponds to random-walk behaviour of the overnight rate - policy inaction on the part of the Federal Reserve.

In order to motivate a possibly nonlinear Taylor rule, Orphanides and Wilcox (2002) postulate a conventional macromodel consisting of an aggregate demand relationship and an expectations-augmented Phillips curve. Their model is summarized as follows:

$$y_t = \rho y_{t-1} - \sigma(r_t - r^*) + u_t \tag{9}$$

$$\pi_t = \pi_t^e + \delta y_t + e_t \tag{10}$$

where y_t is the deviation of output from potential (measured in logarithms) and the parameter ρ is a positive fraction capturing the persistence of the output gap. The deviation of the real interest rate from its steady-state level is denoted by $r_t - r^*$, σ is a positive parameter, and u_t is an aggregate demand shock. Further, π_t and π_t^e are inflation and expected inflation respectively, δ is a positive parameter, and e_t is an aggregate supply shock.

The policy maker is assumed to suffer loss from deviations of inflation from its intermediate target $(\pi_t - \widetilde{\pi})$ and deviations of output from potential according to the following loss function:

$$\mathscr{L} = (\pi_t - \widetilde{\pi})^2 + \xi y_t^2 + \psi\, |y_t| \tag{11}$$

where $\xi \geq 0$, and $\psi \geq 0$. It is further assumed that the intermediate target for inflation is a positive fraction λ of the lagged inflation rate (i.e. $\widetilde{\pi} = \lambda \pi_{t-1}$). The lower the value of λ, the more aggressively the intermediate inflation target is adjusted to its long-run value of zero. It is shown that if neither the demand nor the supply shock can be anticipated, and inflation expectations are static ($\pi_t^e = \pi_{t-1}$) the optimal rule for the policy maker is opportunistic regarding its response to inflation:

$$i_t = \begin{cases} \pi_{t-1} + r^* + \frac{\rho}{\sigma} y_{t-1} + \frac{\delta(1-\lambda)}{\sigma(\delta^2+\xi)}(\pi_{t-1} - \overline{\pi}_{t-1}) & \text{if } \pi_{t-1} > \overline{\pi}_{t-1} \\ \pi_{t-1} + r^* + \frac{\rho}{\sigma} y_{t-1} & \text{if } \underline{\pi}_{t-1} \leq \pi_{t-1} \leq \overline{\pi}_{t-1} \\ \pi_{t-1} + r^* + \frac{\rho}{\sigma} y_{t-1} + \frac{\delta(1-\lambda)}{\sigma(\delta^2+\xi)}(\pi_{t-1} - \underline{\pi}_{t-1}) & \text{if } \pi_{t-1} < \underline{\pi}_{t-1} \end{cases} \tag{12}$$

where $\overline{\pi}_{t-1} = \frac{\psi}{2\delta\alpha(1-\lambda)}$ and $\underline{\pi}_{t-1} = -\frac{\psi}{2\delta(1-\lambda)}$ are an upper and a lower threshold respectively.

The opportunistic monetary policy rule suggests that as long as inflation is within the band defined by a lower and upper threshold $\left[\underline{\pi}_{t-1}, \overline{\pi}_{t-1}\right]$, it does not warrant a policy response. Once inflation exceeds the upper threshold, it triggers an increase in the nominal interest rate intended to bring the real interest rate up toward its long-run level. An analogous argument can be made for rates of inflation that are below the lower threshold. From an empirical standpoint, this model suggests that optimal monetary policy reinforces the mean-reverting properties of the *ex-post* real interest rate when expected inflation lies outside a certain band.

The width of the policy-inaction band is determined by the values of the parameters entering the expressions for the thresholds. For example, a monetary authority with a gradualist approach to disinflation (high λ) will have a relatively wide band of policy inaction. The same conclusion holds for a monetary authority that faces a flat expectations-augmented Phillips curve (low δ) and/or experiences high loss from deviations of output from potential (high ψ).

Following Taylor and Davradakis (2006), we model possibly asymmetric behaviour on the part of the Fed by a three-regime threshold regression model. When the one-year-ahead inflation forecast lies within a certain band described by a lower and an upper threshold (τ_{-}, τ^{-}), the Fed may adopt a less active policy stance. In the extreme, policy inaction may lead to random-walk behaviour of the Fed Funds Rate. On the other hand, when the inflation forecast breaches the upper threshold, the Fed may adjust the Fed Funds Rate more aggressively. Similarly, when the inflation forecast breaches the lower threshold, fears of deflation may force the Fed to act decisively by lowering the Fed Funds rate.

The model is summarized as follows:

$$i_t = \epsilon_t + \begin{cases} \alpha_0 + \alpha_1 i_{t-1} + \alpha_2 \pi^a_{t+3} + \alpha_3 \Delta^a y_{t+3} + \alpha_4 y_{t-1} & \text{if } \pi^a_{t+3} > \tau^{-} \\ \beta_0 + \beta_1 i_{t-1} + \beta_2 \pi^a_{t+3} + \beta_3 \Delta^a y_{t+3} + \beta_4 y_{t-1} & \text{if } \tau_{-} \leq \pi^a_{t+3} \leq \tau^{-} \\ \gamma_0 + \gamma_1 i_{t-1} + \gamma_2 \pi^a_{t+3} + \gamma_3 \Delta^a y_{t+3} + \gamma_4 y_{t-1} & \text{if } \pi^a_{t+3} < \tau_{-} \end{cases} \tag{13}$$

In (13), ϵ_t is a white noise disturbance common across regimes. The thresholds, τ_{-}, τ^{-}, are assumed to be unknown and will be determined endogenously. Within the band $[\tau_{-},\ \tau^{-}]$, policy inaction may lead to random walk behaviour of the Fed Funds rate. The latter is a testable hypothesis and will be investigated in Section 4 of the paper. Further, the Orphanides and Wilcox (2002) optimal policy reaction function implies a symmetric policy response to inflation in the two outer regimes (i.e. it implies the restriction $\alpha = \gamma$, where α and β are parameter vectors).

3 Estimation Strategy

Estimation of (13) is implemented by defining two dummy variables:

$$\begin{aligned} D_t &= 1 \text{ for } \pi^a_{t+3} \geq \tau^- \text{ and } D_t = 0 \text{ otherwise} \\ I_t &= 1 \text{ for } \pi^a_{t+3} \leq \tau^- \text{ and } I_t = 0 \text{ otherwise} \end{aligned}$$

Write:

$$\begin{aligned} i_t = {} & D_t \left(\alpha_0 + \alpha_1 i_{t-1} + \alpha_2 \pi^a_{t+3} + \alpha_3 \Delta^a y_{t+3} + \alpha_4 y_{t-1}\right) \\ & + (1 - I_t - D_t) \left(\beta_0 + \beta_1 i_{t-1} + \beta_2 \pi^a_{t+3} + \beta_3 \Delta^a y_{t+3} + \beta_4 y_{t-1}\right) \\ & + I_t \left(\gamma_0 + \gamma_1 i_{t-1} + \gamma_2 \pi^a_{t+3} + \gamma_3 \Delta^a y_{t+3} + \gamma_4 y_{t-1}\right) + \epsilon_t \end{aligned} \tag{14}$$

Equation (14) is estimated by conditional sequential linear least squares with an adjustment for heteroscedasticity and autocorrelation. For a given pair of values of τ_-, τ^-, estimates of the αs, βs, and γs can be obtained together with the residual variance. The value of τ_-, τ^- that minimizes the residual sum of squares is the LS estimate of the thresholds. Hansen (1996) has shown that a grid search over the interval τ_-, τ^- that minimizes the sum of squared residuals, yields consistent estimates of the thresholds and the model parameters under fairly weak regularity assumptions.

Tests for asymmetric behaviour take place by successively restricting the general model (14) to a two-regime model ($\tau_- = \tau^- = \widehat{\tau}$). The restrictions are tested through a likelihood ratio test

$$LR_T(\tau) = T((\ln(RSS^{restricted}) - \ln(RSS^{unrestricted})) \tag{15}$$

where T is the number of observations, $RSS^{restricted}$ and $RSS^{unrestricted}$ are the residual sum of squares of the restricted and unrestricted models respectively. If a two-regime model cannot be rejected by the data, the null hypothesis of linearity is tested by setting the parameter vectors α and γ equal. An $F - test$ is inappropriate in this case since the threshold is estimated together with other parameters of the model. Hansen (1996) shows how to perform a non-parametric bootstrap procedure in order to derive the empirical significance levels of the test statistics.

The following steps describe the estimation and testing procedures:

1. The data on π^a_{t+3} is sorted in ascending order. The top and bottom 15% of the sorted data is trimmed in order to establish the maximum and minimum values for the thresholds (τ_-, τ^-). This is done in order to insure that at least 30% of the sorted observations will lie outside and inside the middle regime. This way, the estimated model is not unduly influenced by a few important outliers.

2. A grid search is performed within τ_{-}, τ^{-}. The values of the thresholds that minimize the residual sum of squares are selected together with the model estimates of the αs, βs, and γs.

3. In order to implement the testing-down procedure mentioned above, we compute the Likelihood Ratio test statistic (15) conditional on the estimated optimal thresholds.

4. Given $LR_T(\tau)$, we compute a simulated p value (marginal significance level). The null hypothesis, the restricted model, is rejected for a low p value.

The steps involved in the non-parametric bootstrap are outlined in Taylor and Davradakis (2006) and are repeated here for convenience:

(a) Estimate the restricted model using the full sample of T observations as, described above, and store the residuals and the fitted values of the Fed Funds rate

(b) Draw with equal probability and with replacement from the vector of residuals to make up another $T \times 1$ vector of residuals

(c) Add this vector to the vector of fitted values of the Fed Funds rate stored in step (a) to obtain an artificial vector of interest rate observations

(d) Estimate the restricted and unrestricted models using the artificial vector of interest rate observations and construct a simulated test statistic $\widehat{LR}_T(\tau)$

(e) Repeat steps $b-d$ ten thousand times to obtain ten thousand simulated values of $\widehat{LR}_T(\tau)$

The percentage of occurrences where the simulated values of $\widehat{LR}_T(\tau)$ exceed the estimated value of $LR_T(\tau)$ is the empirical marginal significance level of the statistic.

4 Estimation Results

The dataset used in the paper was kindly supplied by Athanasios Orphanides from his *JME* (2003) paper. It spans the period 1969:1-1997:4 and consists of Greenbook data available to the FOMC at the time of its meetings in the middle month of any quarter. The definitions of the variables and the data sources for the extended sample are provided in the Data Appendix.

The encompassing policy reaction function (14) - henceforth referred to as Model M_1 is estimated for the period 1982:3-2003:4. The sample starts roughly at the beginning of Paul Volcker's chairmanship (1982:1-1987:1) and extends well into the Alan Greenspan chairmanship (1987:2-2006:1). The estimation results are reported in Table 1. Inspection of the results revealed that the coefficient estimates for the middle regime are statistically insignificant with the exception of the coefficient of the lagged interest rate that is very close to one. We restricted the middle regime to a random walk as indicated by the following model (M_1')

$$M_1' : i_t = \epsilon_t + \begin{cases} \alpha_0 + \alpha_1 i_{t-1} + \alpha_2 \pi^a_{t+3} + \alpha_3 \Delta^a y_{t+3} + \alpha_4 y_{t-1} & \text{if } \pi^a_{t+3} > \tau^- \\ i_{t-1} & \text{if } \tau_- \leq \pi^a_{t+3} \leq \tau^- \\ \gamma_0 + \gamma_1 i_{t-1} + \gamma_2 \pi^a_{t+3} + \gamma_3 \Delta^a y_{t+3} + \gamma_4 y_{t-1} & \text{if } \pi^a_{t+3} < \tau_- \end{cases} \tag{16}$$

We re-estimated M_1' and formally tested the restriction utilizing the simulated p values reported in Table 2. The restricted model cannot be rejected at the conventional levels of significance ($LR_T(\tau) = 10.90$, $p-$value=0.14) and its estimated thresholds remain unchanged ($\tau_- = 2.8\%$, $\tau^- = 3.9\%$).

We continued with our general-to-specific testing strategy by restricting M_1 to a two-regime model (M_2)

$$M_2 : i_t = \epsilon_t + \begin{cases} \alpha_0 + \alpha_1 i_{t-1} + \alpha_2 \pi^a_{t+3} + \alpha_3 \Delta^a y_{t+3} + \alpha_4 y_{t-1} & \text{if } \pi^a_{t+3} \geq \tau \\ \gamma_0 + \gamma_1 i_{t-1} + \gamma_2 \pi^a_{t+3} + \gamma_3 \Delta^a y_{t+3} + \gamma_4 y_{t-1} & \text{if } \pi^a_{t+3} < \tau \end{cases} \tag{17}$$

M_2 is rejected in the presence of M_1 at the 5 percent significance level ($LR_T(\tau) = 23.69$, $p-$value=0.01) and the estimated single threshold is $\tau = 3.9\%$.

Further restricting M_2 yields the linear model estimated by Orphanides (2003)

$$M_3 : i_t = \alpha_0 + \alpha_1 i_{t-1} + \alpha_2 \pi^a_{t+3} + \alpha_3 \Delta^a y_{t+3} + \alpha_4 y_{t-1} + \epsilon_t \; . \tag{18}$$

M_3 is categorically rejected in the presence of M_1 ($LR_T(\tau) = 61.57, p-$value=0.00). Further, the coefficient estimates of M_3 are very similar to those reported by Orphanides (2003) for the sample period 1982:3-1997:4.

The results so far, suggest that the three-regime model seems to fit the data best. The Federal Funds rate in the middle regime approximates a random walk - consistent with policy inaction by the Fed when expected inflation deviates moderately from its long-run target. When the inflation forecast breaches the upper threshold the Fed Funds rate is raised aggressively to stem inflation ($\alpha_2 = 1.36$ with standard error= 0.49). This estimate implies that a one percent increase in inflation above the upper threshold results in an upward revision of the target for the Fed Funds rate by roughly 2.3 percent ($\alpha_2 = (1 - \alpha_1)(1 + \zeta_\pi)$ or $1 + \zeta_\pi = 1.36/(1 - 0.39) = 2.3$). The latter, clearly raises the real overnight rate and is intended to cool down the economy. When the inflation forecast falls below the lower threshold the Fed Funds rate is lowered in real terms to reflate the economy ($\gamma_2 = 0.61$ with standard error$= 0.17$). This estimate implies that a one percent decline in the inflation forecast below the lower threshold results in a downward revision of the target for the Fed Funds rate by roughly 3 percent ($\gamma_2 = (1 - \gamma_1)(1 + \zeta_\pi)$ or $1 + \zeta_\pi = 0.61/(1 - 0.80) = 3.0$). The last result suggests that the Fed has reacted more vigorously to fears of deflation during the sample period.

Interestingly, in the upper inflation regime the Fed does not respond to the output gap (α_4 is statistically insignificant). By contrast, when the inflation forecast is low, the Fed targets the output gap ($\gamma_4 = 0.23$ with standard error$= 0.04$). The Fed's reaction to inflation in the linear model ($M_3 : \alpha_2 = 0.33$ with standard error$= 0.11$) implies $1+\zeta_\pi = 0.33/(1-0.89) = 3.0$.

The above results suggest that the policy reaction functions in the two outer regimes are different. To test this hypothesis formally, we set $\alpha = \gamma$ (where α and γ are parameter vectors) and perform a Wald test. The results, reported in Table 2, suggest that the symmetry assumption is rejected categorically.

Finally, we tested our preferred model (M_1') for the inflation forecast version of the classic Taylor rule by setting $\alpha_1 = \alpha_3 = \gamma_1 = \gamma_3 = 0$. The Wald test, reported in Table 2, soundly rejects the joint hypothesis of no interest rate smoothing and lack of output growth targeting by policy makers.

5 Concluding Remarks

This paper has employed an updated version of the Greenbook dataset in Orphanides (2003) to extend his estimates of a U.S. policy reaction function to the nonlinear case. A three-regime threshold regression model, motivated by Orphanides and Wilcox (2002), was used for this purpose. In this framework, when inflation forecasts are within a moderate range around the long-run inflation target, the Fed is passive with regard to its interest rate setting policy. Within this range of inaction, the Fed relies on favourable aggregate demand and/or supply shocks to restore inflation to its desired long-run level. Without active Fed policy the Fed Funds Rate is buffeted by random shocks and resembles a random walk. On the contrary, when inflation forecasts veer outside the range of Fed tolerance, aggressive adjustments in the Fed Funds rate take place.

We have followed a general-to-specific approach whereby - through successive restrictions of the general model - we have tried to discover the model that best fits the data. We have found that a two-regime model was rejected in the presence of the three-regime model described above. Similarly, the linear Taylor rule estimated by Orphanides (2003) was categorically rejected in the presence of the three-regime model. Finally, the restriction that was not rejected by the data was that, in the middle regime, the Fed Funds Rate resembles a random walk. Our empirical results seem to support the Orphanides and Wilcox (2002) conjecture that the Fed behaves in an "opportunistic" fashion when it sets the Fed Funds Rate.

A number of issues for further research remain. It is of interest to investigate the sensitivity of the results by using a variation of the threshold model suggested by Enders and Granger (1998). The "momentum" threshold model, as it is called, assumes that the Fed's response depends on the sign of the first difference in the inflation forecast or other threshold variable. Further, it is possible to use longer real-time datasets for the U.S. constructed from vintage data as described in detail in Nikolsko-Rzhevskyy (2009). The use of such techniques allows the estimation of real-time policy reaction functions for a number of countries that do not publish data equivalent to the Greenbook data.

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Data Appendix

The dataset used in the paper was kindly supplied by Athanasios Orphanides from his *JME* (2003) paper. It spans the period 1969:1-1997:4 and consists of Greenbook data available to the FOMC at the time of its meetings in the middle month of any quarter. We have extended the Orphanides dataset to the most recent period for which Greenbook data has been released to the public (2003:4) - as of December, 2009.

The inflation dataset of Greenbook projections was obtained from the Federal Reserve Bank of Philadelphia.[1] The file contains two worksheets: one for real GNP/GDP and one for the GNP/GDP price index. Each column corresponds to a different quarter: QTR0 is the current-quarter projection; QTR1 is one quarter ahead, etc. The projections cover a variety of horizons, depending on what the Fed was forecasting at the time. The forecasts are for quarter-over-quarter rates of change, in annualized percentage points. The inflation forecasts made since 1996:Q4 are also for the chain-weighted price index.

Greenbook forecasts for the output gap at various horizons were also obtained from the Federal Reserve Bank of Philadelphia.[2] The output gap was not included in the Greenbook over the period covered by this data set. The data on the output gap were constructed and used by Board staff in generating wage and inflation forecasts and checking unemployment forecasts. The output gap is defined as the difference between actual and potential output, expressed as a percent of potential output.

Finally, we use the Federal Funds rate released by the Board of Governors of the Federal Reserve System.[3]

The definitions of the variables follow Orphanides (2003) and are as follows:

Variable Definitions

i_t	Federal Funds Rate
P^e_{t+3}	Forecast of the GDP deflator for $t+3$
P_{t-1}	Actual deflator for $t-1$ as reported in t
$\pi^a_{t+3} = (\ln(P^e_{t+3}) - \ln(P_{t-1})) \times 100$	Forecast of annual inflation 3 periods ahead
Y_t	Actual real output
Y^*	Potential real output estimate
$y_t = (\ln(Y_t) - \ln(Y^*)) \times 100$	Output Gap
y^e_{t+3}	3-quarters ahead forecast of y_t
$\Delta^a y_{t+3} = (y^3_{t+3} - y_{t-1}) - (y^{e*}_{t+3} - y^*_{t-1})$	Year-ahead growth forecast relative to potential

[1]This file can be obtained at http://www.philadelphiafed.org/research-and-data/real-time-center/greenbook-data/philadelphia-data-set.cfm.

[2]This file can be obtained at http://www.philadelphiafed.org/research-and-data/real-time-center/greenbook-data/gap-and-financial-data-set.cfm.

[3]Obtained at http://www.federalreserve.gov/releases/h15/data/Monthly/H15_FF_O.txt.

Table 1: Estimation Results

	M_1	$M_1^{'}$	M_2	M_3
α_0	-0.12	-0.12	-0.12	-0.21
	(1.86)	(1.86)	(1.86)	(0.17)
α_1	0.39	0.39	0.39	0.89
	(0.07)	(0.07)	(0.07)	(0.05)
α_2	1.36	1.36	1.36	0.33
	(0.47)	(0.49)	(0.47)	(0.11)
α_3	0.49	0.49	0.49	0.47
	(0.10)	(0.10)	(0.10)	(0.11)
α_4	0.07	0.07	0.07	0.12
	(0.04)	(0.04)	(0.04)	(0.03)
β_0	-1.96			
	(0.98)			
β_1	1.02			
	(0.09)			
β_2	0.52			
	(0.42)			
β_3	0.40			
	(0.23)			
β_4	0.05			
	(0.05)			
γ_0	-0.40	-0.35	-0.32	
	(0.19)	(0.19)	(0.19)	
γ_1	0.80	0.80	0.97	
	(0.05)	(0.06)	(0.05)	
γ_2	0.64	0.61	0.19	
	(0.17)	(0.17)	(0.15)	
γ_3	0.33	0.35	0.33	
	(0.10)	(0.10)	(0.14)	
γ_4	0.22	0.23	0.09	
	(0.04)	(0.04)	(0.03)	
Threshold			3.9%	
Lower	2.8%	2.8%		
Upper	3.9%	3.9%		

Notes: Model M_1 is the unrestricted 3-regime model, $M_1^{'}$ restricts M_1 by imposing a random walk in the middle regime, M_2 is the 2-regime model while M_3 is the linear model. The αs are for the upper regime, βs for the middle regime and the γs for the lower regime. Coefficients indexed with a 0 are for the constant, with a 1 for the coefficients on the lagged interest rate, with a 2 for the coefficients on inflation forecast, π^a_{t+3}, with a 3 for the coefficients on the year-ahead growth forecast, $\Delta^a y_{t+3}$, and with a 4 for the coefficients on the output gap, y_{t-1}. Estimation was performed using least squares and HAC standard errors are in parantheses for the period 1982:3 to 2003:4.

Table 2: Inference Results

M_1 vs $M_1^{'}$	M_1 vs M_2	M_1 vs M_3	Symmetry	Inflation
10.90	23.69	61.57	39.96	329.20
(0.14)	(0.01)	(0.00)	(0.00)	(0.00)

Notes: Model M_1 is the unrestricted 3-regime model, $M_1^{'}$ restricts M_1 by imposing a random walk in the middle regime, M_2 is the 2-regime model while M_3 is the linear model. When comparing models the LR test, as described in the text, is used. The restricted model is always on the right. Column labeled with Symmetry reports a Wald test for the null hypothesis $\alpha = \gamma$ in model $M_1^{'}$. Column labeled Inflation reports a Wald test for the null $\alpha_1 = \alpha_3 = \gamma_1 = \gamma_3 = 0$ in model $M_1^{'}$. p values are in parantheses.