Anticipation and Real Business Cycles

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Abstract

Standard real business cycle (RBC) theory assumes that changes in economic conditions are unanticipated. We argue that upcoming changes are often well anticipated. Employing the RBC methodology to evaluate models when changes in economic conditions are **fully anticipated** provides evidence on the relevance of this alternative. We find that anticipation effects i) reduce the exogenous volatility required for the models to explain output volatility, ii) improves, or leaves unchanged, model predictions for the data moments studied and, iii) can go some way to providing realistic internal propagation mechanisms within theoretical frameworks.

Keywords: Anticipation, Real Business Cycles, Impulse Responses.

JEL: E10, E30, E37.

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1 Introduction

A common feature of real business cycle (RBC) models is that changes in productivity, fiscal policy, monetary conditions, or other "shocks", are assumed to be *unanticipated*. News, or information about future fundamentals unrelated to current conditions, however, may also influence business cycles. For example, regulatory changes and government budgets enacted through legislation are clearly well anticipated prior to taking effect. Also, it is rare for the introduction of a new product, process, or technology not to be accompanied by a series of announcements and analyses. Indeed, the establishment of anticipation and hype for new "revolutionary" products is standard marketing strategy. The timely releases of successive "Windows" operating systems and "Pentium" processors provide ready examples of this.

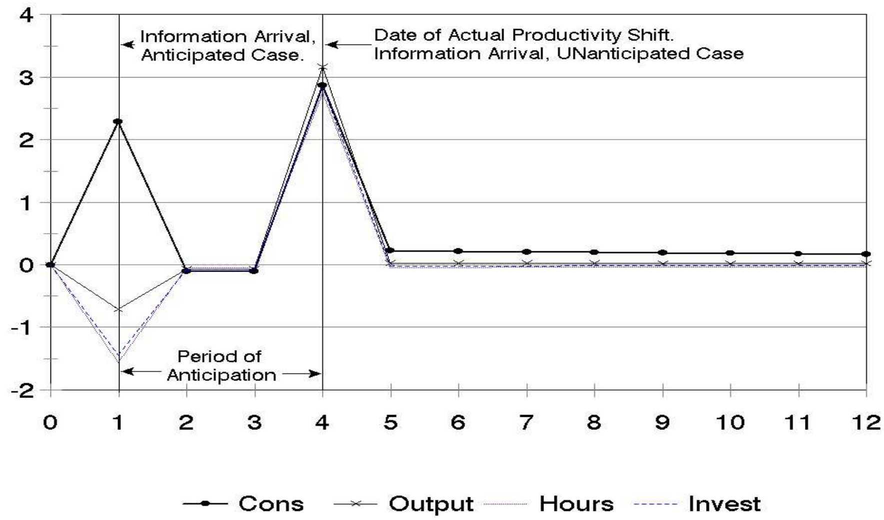
Figure 1 illustrates a standard RBC model's prediction that, *in anticipation* of a future productivity change, consumption will vary *inversely* with output, hours, and investment (eg. see Beaudry and Portier, 2000). Similar predictions result for anticipated changes in government spending (see Appendix A).

These dynamics do not conform with the stylized view of business cycles which emphasizes positive co-movements only. However, such results are readily observed in post-WWII quarterly United States data as is plotted in Appendix A. Of the 187 observations in our sample[2], 54 (28.8%) are consistent with the hypothesis of anticipation effects as characterized by growing consumption in the presence of declining output, hours, and investment, or the reverse.

Since alternatives to the hypothesis of anticipated "shocks" may explain these observations, skepticism may be warranted. However, the standard model of purely unanticipated shocks completely fails to predict the large proportion of observations noted above. In comparison, anticipation provides a simple and

[2] Appendix B details the data series employed throughout the paper.

Figure 1: Model Growth Rates with Anticipated Productivity Improvement.
Growth rate deviations from balanced-growth path, normalized by standard deviations.



plausible explanation for periods of negative correlation *and* periods of positive correlation as we see in the data. It is also likely that the economy experiences a mixture of anticipated and fully unanticipated shocks.

In the remainder of this paper we employ the RBC methodology (Kydland and Prescott (1982), Prescott (1986)) to compare the predictions of several models for data moments and other empirical regularities under alternative assumptions about the degree of anticipation regarding future changes in economic conditions. In our framework agents observe the outcome of conventional stochastic processes some $\tau \in (0, T)$ periods *prior* to their direct effects on the economy. When $\tau > 0$, as illustrated for $\tau = 3$ in Figure 1, the current realization of a stochastic process (the "shock") and the arrival of the implied changes in economic conditions are temporally separated. Agents, fully anticipating the effects of these upcoming changes, can re-optimize during the intervening period.

It may be helpful to contrast our framework with the "time-to-build" structure of Kydland and Prescott (1982) where the arrival of new productive capital some periods in the future is anticipated in the current decision period. In that framework, however, such upcoming changes do not reflect new information but rather are the consequence of prior decisions and are consistent with the optimal trajectory already being undertaken. Here, as elsewhere in the RBC literature, new information which may alter that optimal trajectory coincides with *current* productivity changes.

We find that anticipation magnifies the effects of a given shock and thus allows the model to correctly predict output volatility with significantly lower volatility in the exogenous forcing process. Relative volatilities and correlations predicted by the model are generally improved or left unchanged by the anticipation assumption. We also discuss implications for the internal propagation properties of the model. We document improved impulse responses for hours and output, but find little ability of anticipation per se to generate significant output persistance.

A number of studies employing various mechanisms have also successfully explained aspects of output dynamics, and/or the magnification and propagation of shocks. Andolfatto (1996) and Merz (1995), for example, employ labour-market search. Perli and Sakellaris (1998) examine the role of human capital formation. Gilchrist and William (2000) adopt a specific two-sector technology, as do Boldrin, Christiano, and Fisher (2001). Burnside and Eichenbaum (1996), and King and Rebelo (1998) study factor hoarding and variable capacity utilization, and Aadland (2001) explores temporal aggregation. Our work contributes to this literature by drawing attention to the study of anticipation as an intuitive mechanism that is inherent to *all* dynamic optimization frameworks and can yield these improvements with relative parsimony.

In the next section of the paper we outline a simple base-case RBC model and briefly discuss our simulation approach. Section 3 compares basic moments estimated from the model and from our data sample. In Section 4 we compare estimated impulse-response functions obtained from the model and from the data. In Section 5 we discuss the sensitivity of our results to model parameterization and to several alternative specifications. Section 6 concludes.

2 The Base-Case Model

To facilitate later comparisons, our study starts with a standard single-sector RBC model similar to many found in the literature (see, for example, Christiano and Eichenbaum (CE), 1992; Prescott, 1986). There are no adjustment costs or lags. As in CE, the model is driven by technology and government expenditure shocks. There is a balanced-growth path (BGP) equilibrium characterized by stationary per-capita hours and difference-stationary per-capita output, consumption, and capital stocks. The long-run rate of growth is determined by an exogenous rate of increase in total factor productivity.

2.1 Households and Information Structure

There is a single representative household which maximizes expected utility according to,

$$U = E\{\sum_{t=1}^{\infty} \beta^t u(C_t, (1-l_t)) \mid \Omega_{t+\tau}\}. \tag{1}$$

C_t is period-t consumption, and l_t is labour supplied from a unit endowment of time per period. $\beta < 1$ is the household's subjective rate of time preference. The period utility function is isoelastic over consumption and leisure and is parameterized as:

$$u(C_t, l_t) = \frac{1}{1-\sigma}(C_t^{\varepsilon}(1-l_t)^{(1-\varepsilon)})^{(1-\sigma)} \quad \varepsilon \in (0,1), \quad 0 \le \sigma \neq 1 \tag{2}$$
$$u(C_t, l_t) = ln(C_t^{\varepsilon}(1-l_t)^{(1-\varepsilon)}), \qquad \sigma = 1.$$

$\Omega_{t+\tau}$, $\tau \ge 0$ denotes the household's time-t information set. If $\tau = 0$ we have the standard RBC setup where the household has perfect information about the state of the economy up to and including the current period, but future economic conditions are subject to uncertainty. Since this situation implies that, for example, variations in technologies are never realized in advance of the current decision making period, we refer to this as the *unanticipated case.* Alternatively, if $\tau > 0$, then the household's information set includes knowledge of the state of the economy τ periods beyond the current decision making period, and we refer to this as the *τ-period anticipated case.*

The household's labour supply earns a competitive wage rate, w_t. Households also save directly in capital which is rented out each period for a competitive rate of return r_t, and which follows the usual law of motion, $K_{t+1} = (1-\delta)K_t + I_t$, where $\delta \in (0,1)$ gives the rate of capital depreciation. Lastly, in each period households are subject to a lump-sum tax T_t which is discussed further below. The resulting household income is allocated between consumption goods and capital investments implying the following time-t budget constraint,

$$C_t \le w_t l_t + r_t K_t - I_t - T_t. \tag{3}$$

2.2 Firms and Production Technologies

The production of output, Y_t, is undertaken by a representative firm which employs labour and rents capital to maximize period-by-period profits under a constant returns to scale production technology,

$$Y_t = AS_t K_t^{\alpha} l_t^{1-\alpha}, \tag{4}$$

where, $A > 0$, and $\alpha \in (0, 1)$ are standard production function coefficients. $S_t > 0$ is an exogenous productivity factor or "technology shock" process described further in Section 2.4 below.

2.3 Government

As in CE and Cogley and Nason (1995) the government sector is modelled as a pure resource drain on the economy with no distortionary consumption or investment effects. Each period the budget is balanced using the single non-distorting lump-sum tax. Thus,

$$G_t = T_t,$$

where G_t is the level of government expenditure. Along a BGP government expenditure is assumed to be a constant proportion of aggregate output, $G_t = \gamma Y_t$. Otherwise the evolution of government expenditure is determined by a stochastic process also described further in Section 2.4 below.

2.4 Calibration and Simulation

2.4.1 Stochastic Processes

The exogenous productivity factor, S_t, is assumed to follow a random walk with drift,

$$ln(S_t) = ln(S_{t-1}) + \mu + \xi_{St}, \quad \xi_{St} \sim i.i.d.\ N(0, \sigma_S^2), \tag{5}$$

and government expenditure is modeled as a persistent AR(1) process around the stochastic technology trend implied by Equation 5,

$$ln(G_t) = (1-\rho)\overline{G} + \rho ln(G_{t-1}) + (1-\rho)\tfrac{1}{(1-\alpha)} ln(S_{t-1}) + \xi_{Gt}, \qquad (6)$$
$$\xi_{Gt} \sim i.i.d.\ N(0, \sigma_G^2)$$

where

$$\overline{G} = ln(\gamma) + ln(Y_0) - \frac{1}{(1-\alpha)} ln(S_0).$$

Solving the model's responses to anticipated changes requires evaluation of the expected values of these processes *conditional* on knowledge of specific outcomes for the near future. The following conditional moments apply for the specifications given above,

$$E[ln(S_{t+N}) \mid S_t,\, G_t] = ln(S_t) + N\mu, \qquad (7)$$

$$E[ln(G_{t+N}) \mid S_t,\, G_t] = \rho^N ln(G_t) + \frac{N\mu}{(1-\alpha)} + \frac{(1-\rho)}{(1-\alpha)}(\overline{G} + ln(S_t)) \sum_{i=1}^{N} \rho^{i-1}, \qquad (8)$$

$$Var[ln(S_{t+N}) \mid S_t,\, G_t] = N\sigma_S^2, \qquad (9)$$

$$Var[ln(G_{t+N}) \mid S_t,\, G_t] = N\sigma_s^2/(1-\alpha)^2 + \sigma_G^2 \sum_{i=1}^{N} \rho^{2(i-1)}. \qquad (10)$$

2.4.2 Base-Case Calibration

Time periods in the model are assumed to correspond to quarterly observations. Where possible the calibration is worked so that the model's balanced-growth solution replicates long-run averages for U.S. data. Note that our preference specification implies an intertemporal elasticity of substitution, $IES = 1/(1 - \varepsilon(1-\sigma))$. In calibrating the stochastic technological process we choose μ to generate a quarterly balanced-growth rate of output (g) of 0.42% corresponding to the growth rate of U.S. GDP per worker in our sample[3]. As in Cogley and Nason (1995) we employ the estimates of CE in setting $\gamma = 0.177$, and $\rho = 0.96$ for the government expenditure process, but we set the innovation variances so as to replicate the sample variance of per capita GDP growth (0.0095), in our benchmark *unanticipated case*, while maintaining the relative volatilities of the two shocks[4]. Table 1 provides a summary of the parameter settings and corresponding balanced-growth variable values for our base-case calibration.

Table 1: Parameters and Balanced-Growth Variables: Base-Case Calibration.

Model Parameters
$A = 1.0$, $\alpha = 0.35$, $\delta = 0.024$
$\beta = 0.987$, $\varepsilon = 0.1783$, $\sigma = 4.0$, $IES = 0.6515$
$\mu = 0.0027$, $\sigma_S = 0.0077$, $\gamma = 0.177$, $\rho = 0.96$, $\sigma_G = 0.0138$
Balanced-Growth Variable Values
$g = 0.0042$, $l = 0.191$
$G/Y = 0.177$, $Kinv/Y = 0.226$, $r - \delta = 0.0197$

2.4.3 Simulating the Model with Anticipated Technological Change

We simulate the model by the deterministic extended path method due to Fair and Taylor (1984). Gagnon (1990) and Taylor and Uhlig (1990) discuss the use of this method in solving non-linear stochastic growth models. They find a high degree of accuracy with this approach relative to grid methods. Appendix C outlines this method in further detail within the context of the current model and provides other specifics regarding our simulations.

[3] We divide by the labour force here rather than population to factor out increases in output per-capita due to the significant increases in participation rates over the sample.

[4] $\sigma_S/\sigma_G(1-\alpha) = 0.86$, is the appropriate figure for comparison with Cogley and Nason (1995).

3 Base-case Simulation Results

Table 2 presents common RBC statistics calculated from the U.S. data, and from simulations of our base-case model under three alternative assumptions. The first simulation assumes that technological and government expenditure changes are unanticipated, and provides our benchmark case[5]. In the second and third simulations we instead assume that these changes are fully anticipated 1 quarter and 3 quarters repectively in advance of their actual arrivals[6].

Table 2: Volatilities and Cross-Correlations: U.S. Data, Benchmark Unanticipated, 1-quarter, and 3-quarter Anticipated Base-Case models. Logged and Hodrick-Prescott Filtered Data.

	U.S. Data	Unanticipated	1-qtr Anticipated	3-qtr Anticipated
σ_{g_Y}	0.0095	0.0095	0.0128	0.0127
σ_Y	0.0162	0.012	0.0134	0.0149
σ_C/σ_Y	0.523	0.667	0.524	0.363
σ_I/σ_Y	2.40	1.712	2.64	3.224
σ_l/σ_Y	0.783	0.344	0.531	0.645
σ_l/σ_w	1.568	0.4768	0.906	1.453
σ_G/σ_Y	1.198	1.935	1.745	1.559
$\rho(C,Y)$	0.787	0.948	0.803	0.681
$\rho(I,Y)$	0.924	0.956	0.895	0.925
$\rho(l,Y)$	0.871	0.865	0.881	0.943
$\rho(l,w)$	0.176	0.722	0.594	0.668
$\rho_1(g_Y)$	0.332	0.0076	-0.223	-0.008
$\rho_1(g_C)$	0.246	0.0358	0.496	0.009

Notation: σ_x gives the standard deviation of variable x. $\rho(x,y)$ gives the correlation of variables x and y. $\rho_1(x)$ gives the first-order autocorrelation of variable x. Y denotes aggregate output, and g_Y its growth rate. C denotes consumption, and g_C its growth rate. I denotes capital investment, l total hours. w denotes average labour productivity which we calculate as Y/l.

Anticipation raises the variance of output (σ_Y) and the variance of output growth (σ_{g_Y}) for given shock processes. This magnification of shocks in the

[5] All simulated time-series contain 187 observations corresponding to our sample period of 1954:1 to 2000:3. All model estimates reported are averages over $R = 1000$ replications.

[6] Beaudry and Portier (2000) estimate anticipation period lengths of between 2 and 4 quarters using a simulated method of moments.

model implies a lessor reliance on exogenous volatility to capture the data and thus addresses an important criticism of RBC theory.

The added volatility is due to the separation of income and substitution effects across time that is allowed for by anticipation. As illustrated in Figure 1, income effects arise first with the revelation of new information about changes in future economic conditions. For example, anticipated higher future productivity raises agent's expected incomes and leads to increased current consumption and leisure. Substitution effects do not occur until the changes actually take place as, for example, when the actual implementation of new technologies leads to increased wages and hours worked. In contrast, if a change is *un*anticipated both effects arise simultaneously and, working in opposite directions, they tend to cancel out resulting in smaller responses overall.

Likewise, anticipation also raises the relative volatility of hours-to-output (σ_l/σ_Y) bringing the model's predictions closer in line with a feature of the data that has proven difficult to match in simple RBC frameworks. These same anticipation effects on labour supplies raise the relative volatility of hours to average productivity (σ_l/σ_w) again bringing the model closer in line with the data. For government spending shocks (see Figure 9, in Appendix A), where changes in hours are the only direct influence on output, these effects decrease the relative volatility of government spending-to-output (σ_G/σ_Y).

Anticipation enhances consumption smoothing reducing the relative volatility of consumption to output (σ_C/σ_Y). This requires a shifting of variability onto investment and the relative volatility of investment-to-output (σ_I/σ_Y) rises under anticipation so as to more closely match the data.

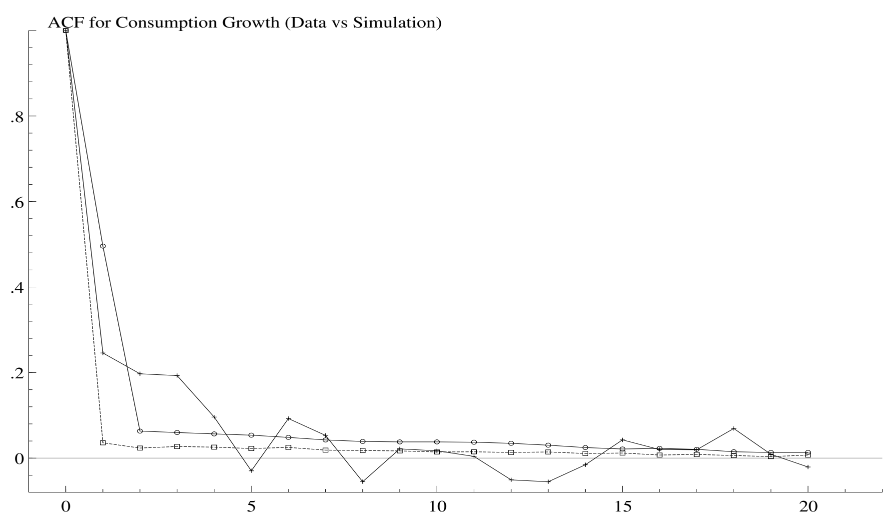
The negative co-movements of consumption and output in the anticipation period offset the positive co-movements of these variables that arise when changes actually take effect. Thus, the near perfect correlation between consumption and output ($\rho(C, Y)$) which characterizes many RBC models is significantly reduced under anticipation, and the model prediction conforms more closely with the empirical value. The correlation of investment and output ($\rho(I, Y)$) is also reduced in the anticipated case coming closer to, but now less than, the data in the 1-quarter case and precisely matching the data in the 3-quarter case.

Given the increased incidence of hours variations driving output variations under anticipation, it is not surprising that the model predicts a higher correlation of hours and output ($\rho(l, Y)$) in the anticipated cases. The model continues to fit the data well in this regard for the 1-quarter case but becomes worse with the longer period of anticipation.

In general the autocorrelations of output and consumption growth are sensitive to the length of anticipation period assumed. Refering to the example of Figure 1, when anticipation occurs only 1 period (quarter) ahead, the two peaks of consumption growth and the trough followed by a peak of output growth, arise in adjacent time periods. This results in strong first-order autocorrelations, positive for consumption growth and *negative* for output growth. When anticipation occurs more than one period ahead these growth rate peaks (or trough and peak) are temporally separated, and first-order autocorrelations are replaced by higher orders commensurate with the number of periods of anticipation assumed.

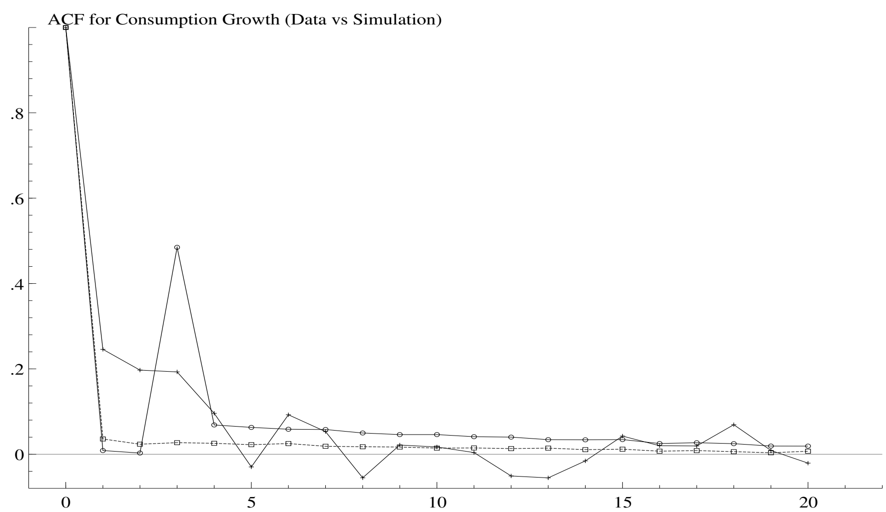
This is further illustrated in Figures 2-5 which plot estimated consumption-growth and output-growth autocorrelation functions from the model under alternative periods of anticipation, and from our data estimates. Note how strong first-order autocorrelations in the 1-quarter anticipated case are replaced by strong third-order autocorrelations in the 3-quarter anticipated case. These autocorrelation properties of the model are artifacts of the simplifying assumption that anticipation periods are of uniform length for all events. It is intuitive, and apparent from our results, that variable periods of anticipation could improve the model's predictions. For example, mixing the anticipation effects in Figures 2 and 3 for the autocorrelation of consumption growth could produce average results more closely approximating the data. Likewise, the changing sign of prediction errors seen in Table 2 between the benchmark unanticipated and anticipated cases suggests that a mixture of unanticipated and anticipated changes are reflected in the data. We turn next to the estimation of some alternative measures of this model's dynamic properties.

Figure 2: Estimated *Consumption-Growth* Autocorrelation Functions; Data, benchmark unanticipated, and **1-quarter anticipated** base-case models.



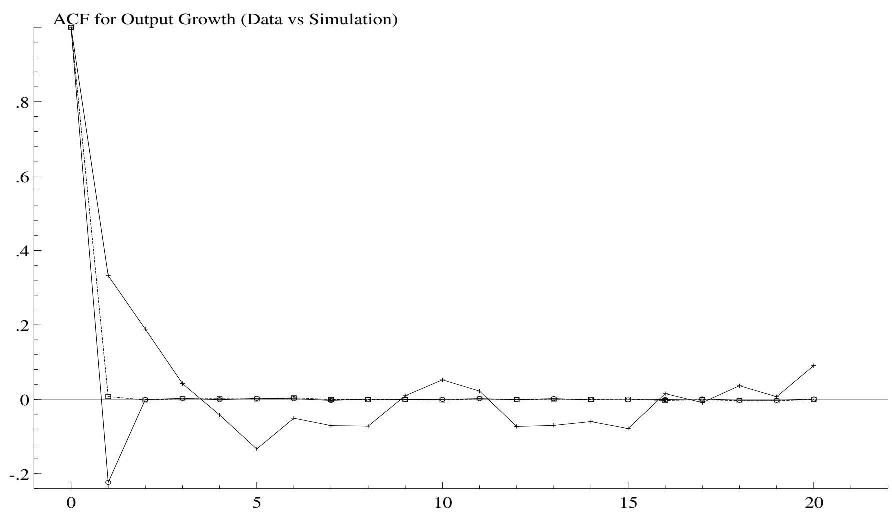
+ U.S. data, □ Unanticipated, ○ 1-qtr Anticipated

Figure 3: Estimated *Consumption-Growth* Autocorrelation Functions; Data, benchmark unanticipated, and **3-quarter anticipated** base-case models.



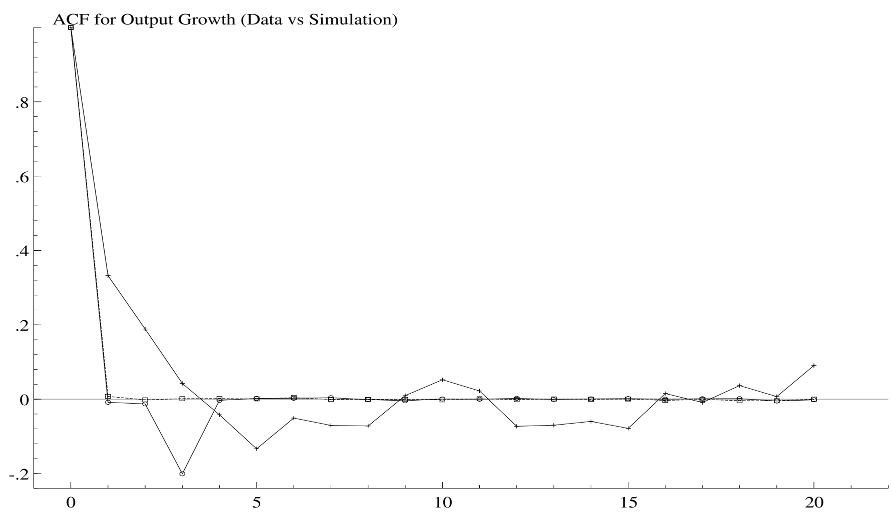
+ U.S. data, □ Unanticipated, ○ 3-qtr Anticipated

Figure 4: Estimated *Output-Growth* Autocorrelation Functions; Data, benchmark unanticipated, and **1-quarter anticipated** base-case models.



+ U.S. data, □ Unanticipated, ○ 1-qtr Anticipated

Figure 5: Estimated *Output-Growth* Autocorrelation Functions; Data, benchmark unanticipated, and **3-quarter anticipated** base-case models.



+ U.S. data, □ Unanticipated, ○ 3-qtr Anticipated

4 Impulse-Response Functions

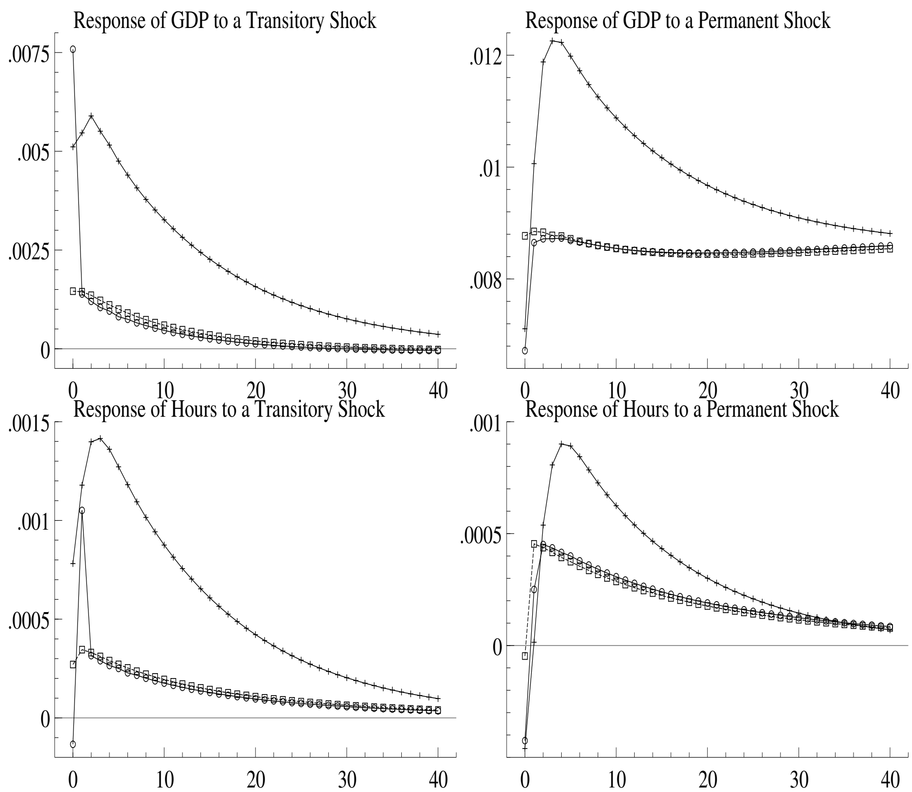
Following Cogley and Nason (1995) we use the method of Blanchard and Quah (1989) on our simulated data to estimate the model's impulse-respnse functions for output and hours. Figures 6 and 7 plot the impulse response functions estimated in the 1-quarter and 3-quarter anticipated cases respectively together with those obtained from the benchmark unanticipated case and with the empirical estimates.

As is well known (see, for example, Blanchard and Quah (1989)) the data for both GDP and hours show significant transitory and permanent responses that are characteristically "hump-shaped". Standard RBC models, as in our *un*anticipated case, show small responses lacking any significant mean reversion (Cogley and Nason (1995)). In contrast, allowing 1-quarter of anticipation adds large initial responses to each of the estimated functions. This results in a peaked or non-monotonic character of the responses indicating some mean reversion, except for the case of the transitory response of GDP. Also of note is the initial negative response generated in the permanent hours function which closely resembles the data. Clearly though the model continues to fall short of explaining the data, particularly with the failure to generate significant responses at longer lags.

Allowing 3-quarters of anticipation, introduces larger and more sustained transitory responses for both output and hours. Permanent responses, although small relative to the unanticipated case, are distinctly hump-shaped. Thus, anticipation can add sizable, prolonged, and non-monotonic features to impulse-response functions estimated from the model.

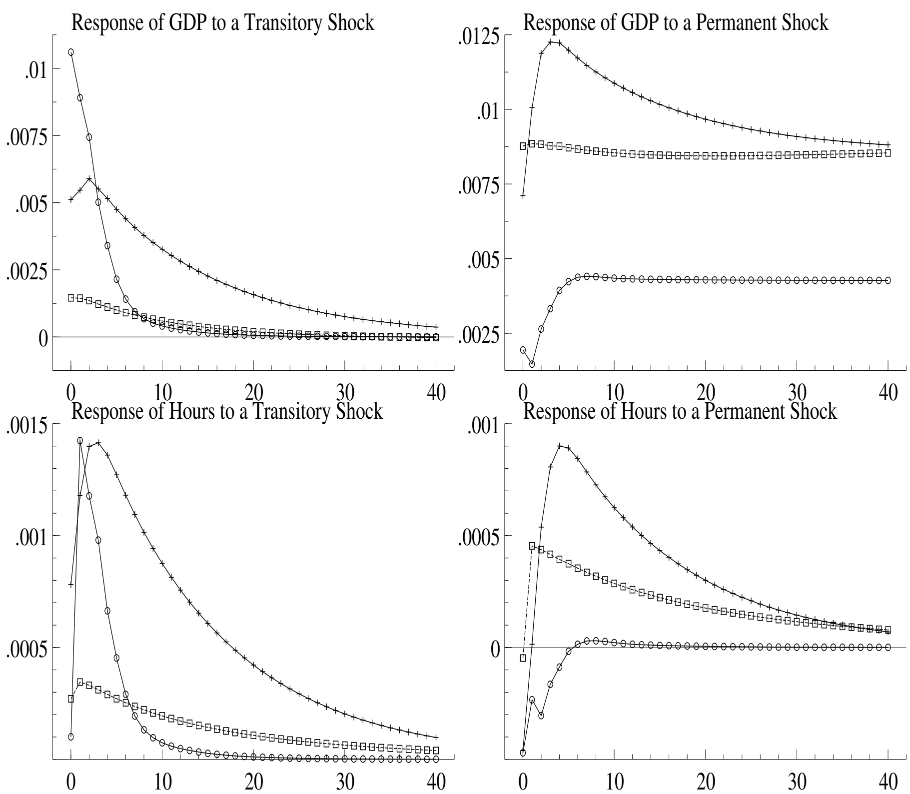
However, table 3 reports test statistics providing a quantitative test of whether a given function estimated from the simulations (approximated by its first eight lags) is a reasonable approximation of the corresponding function obtained from

Figure 6: Estimated Impulse-Response Functions; Data, benchmark unanticipated, and **1-quarter anticipated** base-case models.



+ U.S. data, □ Unanticipated, ○ 1-qtr Anticipated

Figure 7: Estimated Impulse-Response Functions; Data, benchmark unanticipated, and **3-quarter anticipated** base-case models.



+ U.S. data, □ Unanticipated, ○ 3-qtr Anticipated

the data. In all cases the model is strongly rejected. Thus, the evidence in favour of anticipation as an internal propagation mechanism remains largely qualitative.

Table 3: Test Statisitics for Base-Case Model Impulse-Response Functions.

Statistic	Unanticipated	1-qtr Anticipated	3-qtr Anticipated
Q_{Yt}	22.85 (0.0036)	52.25 (0.0)	27.6 (0.0006)
Q_{Yp}	65.36 (0.0)	18.51 (0.017)	50.9 (0.0)
Q_{lt}	612.0 (0.0)	1669.0 (0.0)	60.7 (0.0)
Q_{lp}	725.6 (0.0)	90.3 (0.0)	123.3 (0.0)

All statistics are $\chi^2(8)$, see appendix D for details. P-values in parentheses. Yt and lt refer to the transitory components of output and hours respectively, Yp and lp refer to the permanent components.

5 Sensitivity.

5.1 Intertemporal Elasticity of Substituion (IES).

This section adopts the calibration employed by CE which differs mainly in assuming $IES = 1.0$ versus $IES = 0.6515$ in our base-case. Table 4 below summarizes the parameter and balanced-growth values under this calibration.

Table 4: Parameters and Balanced-Growth Variables: CE Calibration.

Model Parameters.
$A = 1.0$, $\alpha = 0.344$, $\delta = 0.021$
$\beta = 0.9926$, $\varepsilon = 0.2032$, $\sigma = 1.0$, $IES = 1.0$
$\mu = 0.0026$, $\sigma_S = 0.0068$, $\gamma = 0.177$, $\rho = 0.96$, $\sigma_G = 0.0121$
Balanced-Growth Variable Values
$g = 0.0040$, $l = 0.2307$ (315.85/1369)
$G/Y = 0.177$, $Kinv/Y = 0.265$, $r - \delta = 0.0114$

Table 5 presents the results from these simulations which show qualitatively identical effects to those presented for the base-case model. Quantitatively, however, the effects of anticipation on output volatility and output-growth volatility are *larger* with the higher IES. This reflects an enhanced ability to substitute across time in anticipation of future changes. This effect also shows up in the higher relative volatilities of hours worked, and underlies reduced consumption-output correlations. In general anticipation provides less improvement of the model predicitions under this calibration

Table 5: Volatilities and Cross-Correlations: U.S. Data, Benchmark Unanticipated, 1-quarter, and 3-quarter Anticipated cases. CE calibration. Logged and Hodrick-Prescott Filtered Data.

	U.S. Data	Unanticipated	1-qtr Anticipated	3-qtr Anticipated
σ_{g_Y}	0.0095	0.0095	0.0136	0.0135
σ_Y	0.0162	0.0121	0.0138	0.0158
σ_C/σ_Y	0.523	0.484	0.369	0.253
σ_I/σ_Y	2.40	2.071	2.84	3.31
σ_l/σ_Y	0.783	0.468	0.66	0.776
σ_l/σ_w	1.568	0.80	1.479	2.49
σ_G/σ_Y	1.198	1.69	1.48	1.29
$\rho(C, Y)$	0.787	0.892	0.527	0.084
$\rho(I, Y)$	0.924	0.98	0.945	0.961
$\rho(l, Y)$	0.871	0.933	0.932	0.969
$\rho(l, w)$	0.176	0.792	0.595	0.611
$\rho_1(g_Y)$	0.332	0.0034	-0.25	-0.0095
$\rho_1(g_C)$	0.246	0.079	0.11	0.026

Notation: σ_x gives the standard deviation of variable x. $\rho(x, y)$ gives the correlation of variables x and y. $\rho_1(x)$ gives the first-order autocorrelation of variable x. Y denotes aggregate output, and g_Y its growth rate. C denotes consumption, and g_C its growth rate. I denotes capital investment, l total hours. w denotes average labour productivity which we calculate as Y/l.

It is clear that the calibration which best captures the data, and likewise the empirical evidence supporting a particular calibration, are not independent of the anticipation assumptions maintained in the model. This implies some difficulty in interpreting empirical work that identifies particular model parameters without accounting for anticipation effects. The pioneering work of Christiano and Eichenbaum (1992) represents just one example of where this concern may apply. More generally, the identification and measurement of fluctuations in total factor productivity may rely on identification of anticipation effects. These issues fall outside of the scope of this paper, however, and remain for future research.

5.2 Shock Persistance.

Here we examine the implications of the degree of persistence in productivity shocks for our results. For brevity, this section discusses only the main points from the detailed presentation in our working paper (Lamarche and Love, 2004).

The production function specification in the model is modified to the following,

$$Z_t = A_t S_t K_t^{\alpha} l_t^{1-\alpha}, \quad where, \quad A_t = (1+\mu)A_{t-1} + \xi_t. \tag{11}$$

As before S_t is a random productivity shock but is now specified as,

$$\begin{gathered} lnS_t = z_t - \sigma_{\xi}^2/2(1-\nu^2), \quad z_t = \nu z_{t-1} + \xi_t, \\ and, \quad \xi_t \sim i.i.d.\, N(0, \sigma_{\xi}^2)\, \forall t. \end{gathered} \tag{12}$$

These assumptions imply that $E[S_t] = 1$ and the model has a well defined BGP[7]. Further, for small μ and in the limit as $\nu \rightarrow 1$ this model is equivalent to that presented in Section 2 above where we assumed a random walk with drift.

The model was simulated for alternative choices of the persistence parameter with corresponding adjustments to σ_ξ so as to maintain $\sigma_{g_y} = 0.0095$. In all other regards the calibration remained the same as for our base-case model.

The model results with $\nu = 0.99$ are identical to those for the base-case random walk model. As the level of persistence in the shock process diminishes,

[7]This specification for the shock process is adopted from Jones, Manuelli, and Siu (2000).

however, the differences between the benchmark unanticipated case and the anticipated cases narrows. This effect is consistent across all results and continues until, for the $\nu = 0.0$ case, there is no difference between the benchmark unanticipated and anticipated cases.

Intuitively, the income or wealth effects from a given change are larger the longer lasting the effects of those changes. For short-lived changes then income effects are small and little benefit is available from the intertemporal substitution possibilities allowed for during a period of anticipation. Essentially the only difference between the anticipated and unanticipated cases when $\nu = 0.0$ is simply the timing of the productivity change, and thus of any observed substitution effects, so that these two cases become indistinguishable statistically.

5.3 An Endogenous Growth Model (EGM)

The models examined so far specify long-run growth as an exogenous process. Jones *et al* (2000) provide a detailed comparison of the business cycle predictions of exogenous growth RBC models with a two-sector endogenous growth model (EGM). They highlight the EGM's persistence properties. Also, allowing for intersectoral substitution may be important for the effects of anticipation. For brevity, we again make reference to our working paper for a detailed presentation and discuss here only the main points.

5.3.1 Model Specification

As in Lucas (1988) there are two sectors in the economy: the human-capital sector and the final-goods sector[8]. We assume the same utility and information structure for the representative household as in Section 2.1 above. Available time though is allocated between leisure, hours supplied for employment in

[8]This class of models has been studied extensively in the growth literature. See for example, Rebello (1991), and King and Rebello (1990). Gomme (1993) examined a monetary version of this model in a RBC context.

the final-goods sector (l_{Kt}), and hours employed in human-capital formation (l_{Ht}). Human capital (H_t) is embodied within individuals to provide a supply of "effective" labour ($H_t(l_{Kt} + l_{Ht})$). Each sector employs this effective labour and physical capital under constant returns to scale according to,

$$Y_t = A_K S_{Kt} (\theta_t K_t)^{\alpha} (l_{Kt} H_t)^{1-\alpha}, \tag{13}$$

$$X_t = A_H S_{Ht} ((1-\theta_t) K_t)^{\gamma} (l_{Ht} H_t)^{1-\gamma}. \tag{14}$$

Y_t is final-goods production, X_t is human-capital production, θ_t is the proportion of the physical-capital stock allocated to final-goods production, $A_K > 0$, $A_H > 0$, $\alpha \in (0,1)$, and $\gamma \in (0,1)$ are standard production function coefficients, and $S_{Kt} > 0$ and $S_{Ht} > 0$ are exogenous productivity shocks. For simplicity we assume symmetric shocks in each period so that $S_{Kt} = S_{Ht} = S_t \; \forall t$, and S_t follows the process given by Equation 12 in Section 5.2 above[9].

Aggregate economic output from the model is calculated as,

$$Z_t = Y_t + q_t X_t, \tag{15}$$

where q_t gives the relative price of human-capital in terms of final-goods output.

5.3.2 Symmetric Sectors.

Assuming symmetry between production sectors (i.e. $A_K = A_H$, $\alpha = \gamma = 0.35$, $\delta_K = \delta_H = 0.024$) amounts to assuming a single-sector model of homogeneous output ($q_t = 1$) and therefore differs from the model of Section 5.2 only in regards to the endogenous versus exogenous growth assumption. Endogenizing the growth process in itself has little impact on the effects of anticipation for

[9] As opposed to the model of Section 5.2, however, these shocks imply permanent effects in the EGM.

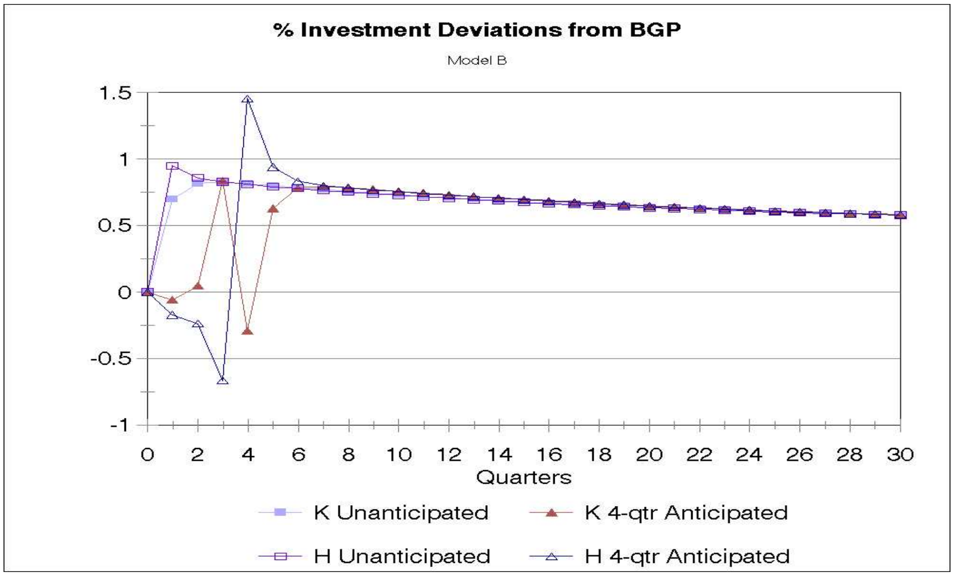
the business cycle predictions of this class of models. The results are perfectly analogous to those for the model of Section 5.2. The effects of anticipation continue to diminish with the degree of persistence in the shock process.

5.3.3 Asymmetric Sectors

In the perfectly symmetric EGM the critical ratio of human-to-physical capital is a constant and transitional dynamic responses to changes in economic conditions transpire in a single period. There is no intersectoral shifting of resources ruling out this avenue for the effects of anticipation. To address this we reduce the share of physical-capital in the human-capital sector ($\gamma = 0.34$) by a small margin such that the capital-income share remains approximately 0.35. In a second alternative we reduce the rate of depreciation on human capital relative to the rate on physical capital ($\delta_K = 0.024$, $\delta_H = 0.018$). In both of these cases we fix the persistence of shocks $\nu = 0.95$.

For hours, consumption, and output, transition paths under anticipation remain qualitatively identical to those in the base-case one-sector model. However, since the ratio of human-to-physical capital is now variable, the investment paths for human and physical-capital show markedly different behaviours. Figure 8 displays the percentage deviations of physical and human-capital investment from a deterministic BGP in both the unanticipated and 3-quarter anticipated cases of the asymmetric EGM model given a positive 1/2 percent productivity shock. Human-capital is heavily weighted in production relative to physical-capital and, in anticipation of future increases in total factor productivity, the least cost way to raise current consumption is to defer human-capital accumulation in the current relatively low productivity period until the improved technology arrives. Thus, in the anticipated case physical-capital investment is low on the arrival date (start of quarter 4) while human-capital investment is high. This results in a low prediction for the correlation of output and physical-

Figure 8: Transitional Dynamic Responses of Capital Investments: EGM.



capital investment.

For the other moments studied we again find the same pattern of results as previously. Also, estimated impulse-response and autocorrelation functions for these asymmetric cases show almost no variation relative to the symmetric sector model. Allowing for the intersectoral reallocations of resources does not alter the relative effects of anticipation in these regards.

5.3.4 Alternative Measurements of the Human Capital Sector

Our working paper also extensively modifies our base-case and asymmetric sectors EGM models to address the mismeasurement (or non-measurment) of human capital as discussed in Jones *et al.* (2000). In the asymmetric-sector cases variations on previous results are observed. Most significantly, the asymmetric cases show substantial positive autocorrelation of output growth under anticipation, improving the model's predictions for the autocorrelation function of output growth.

In regards to estimated impulse-response functions, the results are more dramatic than in the previous models studied, reinforcing the basic message. Anticipation effects significantly improve predictions with respect to the dynamic properties of output and hours. A large transitory component in the estimated response functions arises under anticipation where little is present when changes arrive without warning. Additionally, the response functions often display the typical hump-shape found in the data.

6 Conclusions

We argue that changes in economic conditions are often well anticipated rather than being fully unanticipated as implicitly assumed in standard RBC theory. Further, the behaviour of aggregate consumption, output, investment, and hours are shown to be consistent with anticipation effects as predicted by a prototypical RBC model. Employing a simple methodology to represent the anticipation of future events we explore the implications of fully anticipated government expenditure and technological changes for the predicitons of RBC models.

Our simulation results show first that anticipation can improve almost all of the models' predictions for the data moments studied. Second, anticipation magnifies the effects of a given shock allowing the models to correctly predict the volatility of output growth with significantly less volatility assumed in the exogenous shock processes. Third, we find improvements in the models' predictions for estimated impulse-response functions of output-growth and hours. In particular, transitory responses to a shock are significantly increased under anticipation and hump-shaped responses similar to those in the U.S. data are common. This suggests that anticipation effects can go some way in providing realistic internal propagation mechanisms within theoretical economic models and to improving our understanding of various economic phenomena including business cycles. This makes sense given the degree to which human behaviour is linked with the ability to anticipate events and consequences.

Several avenues for future research are suggested by our work. First, given a model specification, the calibration which best fits the data is not independent of the anticipation assumptions maintained. Likewise, the empirical implementation of particular models, and the identification of total factor productivity in general, will be sensitive to anticipation assumptions. Empirical methodologies need to be developed to account for anticipation effects.

Second, we assume that anticipation periods are of uniform length for all events. This artificial structure particularly shows up in odd predictions for autocorrelation functions. Intuitively, actual economies are likely characterized by combinations of both anticipated and unanticipated changes. We see the study of richer structures, with the extent of anticipation depending on the particular shock or type of shock, as a promising research avenue.

Finally, although anticipation qualitatively improves the predictions for the propagation of shocks in the models employed here, a large quantitative failure remains. Test statistics consistently reject impulse-reponse functions estimated from our simulated data, with or without anticipation effects. Thus, the evidence presented here in favour of anticipation as an empirically relevant internal propagation mechanism is largely qualitative. Important anticipation effects are likely generic to all models of intertemporal optimization, however. As such, the exploration of anticipation effects in frameworks with enhanced shock propagation relative to the basic models studied here likely may further improve predictions in this regard.

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Appendix A. Additional Figures and Data Plots

Figure 9: Model Growth Rates with Anticipated Gov't Expenditure Rise.
Growth rate deviations from balanced-growth path, normalized by standard deviations.

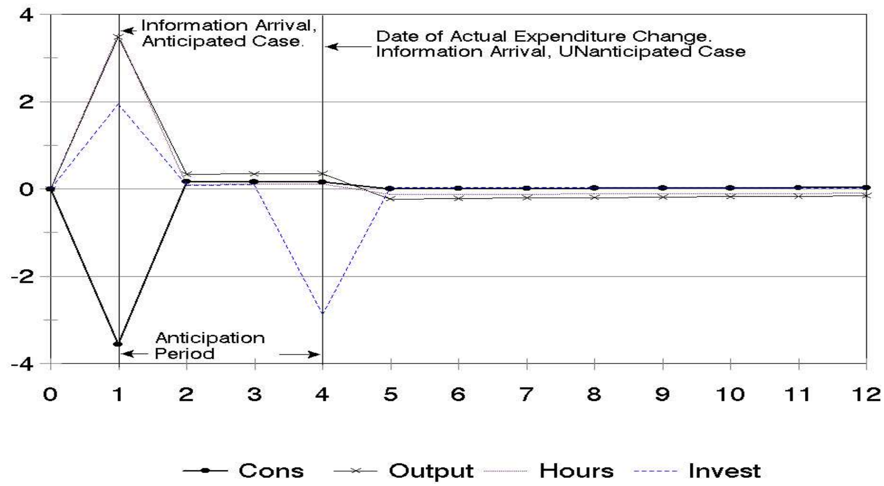


Figure 10: U.S. Data, Growth Rates, 1954-1966.
First-differences of logged and HP-filtered data, normalized by sample standard deviations.

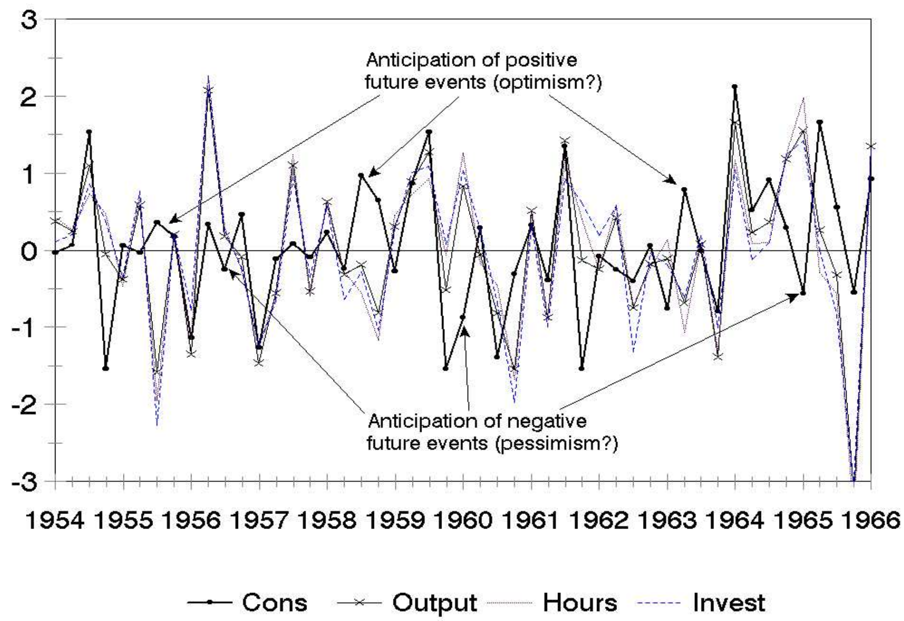


Figure 11: U.S. Data, Growth Rates, 1966-1979

First-differences of logged and HP-filtered data, normalized by sample standard deviations.

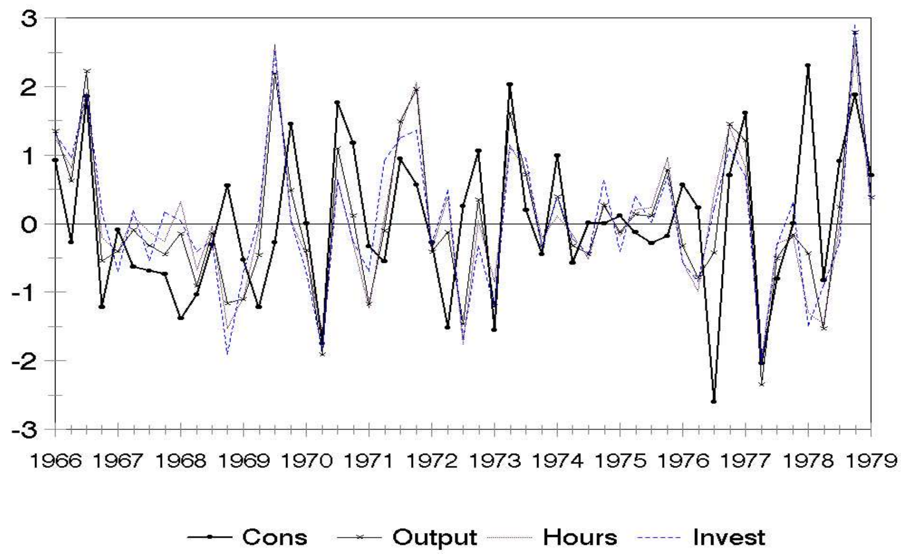


Figure 12: U.S. Data, Growth Rates, 1979-1990

First-differences of logged and HP-filtered data, normalized by sample standard deviations.

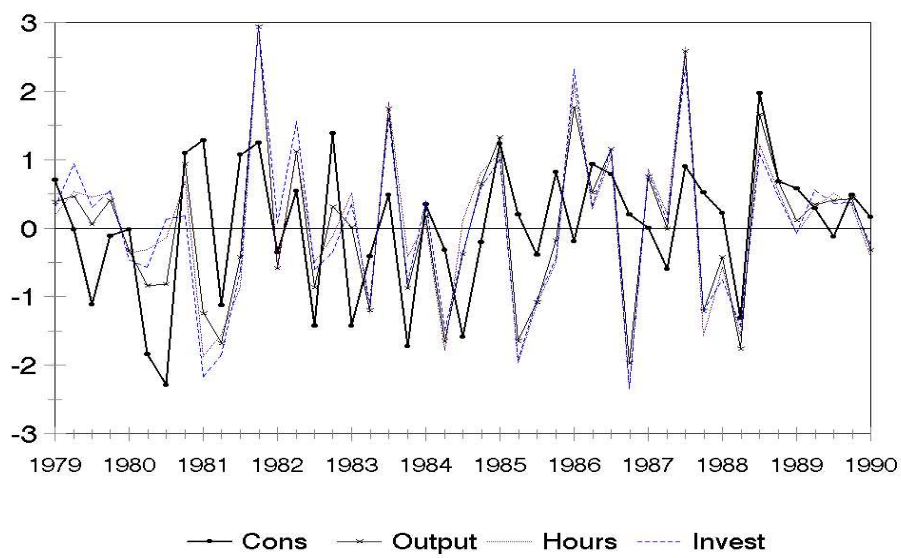
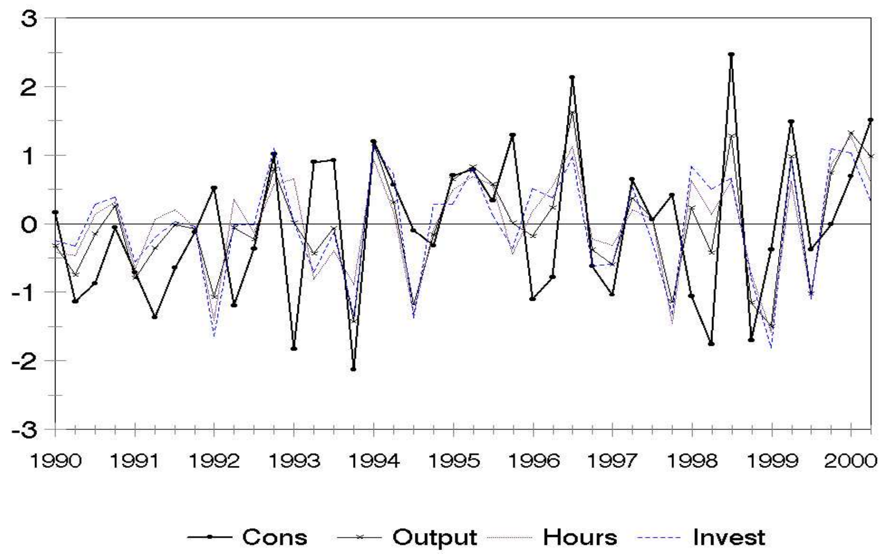


Figure 13: U.S. Data, Growth Rates, 1990-2000:2

First-differences of logged and HP-filtered data, normalized by sample standard deviations.



Appendix B. Data

U.S. quarterly data series employed covered the period 1954:1 to 2000:3. Series denoted in italics where obtained from the National Income and Product Accounts data matrix (*NIPAQ*) downloaded from the EconData web-site at the University of Maryland. Series denoted in noun-style type where obtained from the Federal Reserve Economic Database (FRED) and were aggregated from monthly to quarterly. Where applicable all series were deflated by the implicit price deflator (*d0104*), and were divided by the civilian non-institutional population 16 years of age and older (CNP16OV) to obtain per-capita values.

Output figures are gross domestic product (*n0101*). Consumption is measured as the sum of personal consumption expenditures on non-durables (*c0206*) and services (*c0213)*. Government expenditure was measured as the sum of Federal government defense expenditures (*g0704*), Federal government non-defense expenditures (*g0715*), and State and Local government expenditures (*g0728*). Capital investment was measured by the sum of, private fixed investment (*v0401*), Federal government defense investment (*g0711*), Federal government non-defense investment (*g0724*), State and Local government investment (*g0735*), and personal consumption expenditures on durables *(c0202)*.

Our measure of "hours worked" accords closely with the Citibase "Lhours" series which was not used as it ends at the last month of 1993. Thus we constructed, $hours = avg.hrs. \times employment\ rate \times participation\ rate$, that is, $hours = avg.hrs. \times employment \div population$, where avg.hrs. was given by, average weekly hours of production workers (BLS national employment, hours, and earnings series EEU005 annual figures extrapolated to quarterly) for the period 1954:0 to 1963:4, and by average weekly hours of non-agricultural workers (*awhnonag*) for the period 1964:1 to 2000:3 (monthly data averaged to quarterly). The later series was employed as the former is seasonally unadjusted after 1964. In any case, all of our results appear robust to this choice. Employment was measured by civilian employment 16 years of age and older (CE16OV), and population was again (CNP16OV). Finally, the labour force was measured by the civilian labour force 16 years of age and older (CLF16OV).

Appendix C. Numerical Simulation

For concreteness, but without lack of generality, we discuss our simulation approach with specific reference to the model of Section 2. At the beginning of any time period t, the economy's initial conditions are predetermined by the current value of the state variable K_t. Also at the beginning of this period we assume that agent's realize the time $t+\tau$ outcome of the stochastic processes $\{S_t, G_t\}$, for some $\tau \geq 0$. Equivalently, at each point in time we have an assumption of perfect foresight for the future period $\{t, ..., t+\tau\}$, and the future sequence of technology and government expenditure variables is therefore known for this sub-period. Forecasts for the values of technology and government expenditure for time periods $\{t+\tau+1, ..., T\}$ are then constructed conditional on this information, and according to agents' knowledge of the evolution of the stochastic processes $\{S_t, G_t\}$ as given by equations 5 to 10. T gives the possibly infinite length of agents' forecast horizon. A rational expectations forecast for the entire future sequence of technology and government expenditure variables is thus made and a solution for the economy's optimal transitional response in light of this sequence can be calculated by employing standard methods[10]. This expected transition path yields observations for the time t choice variables of the model which determine the time t+1 state variable values. These state variables in turn represent the time t+1 initial conditions for the economy when a new outcome of the stochastic process $\{S_t, G_t\}$ is realized and thus the basis

[10]We employ a two-point boundary solution method based on exact specifications of the model's dynamic equations system. As discussed by Fair and Taylor (1984), this imposes the model's terminal conditions improving the efficiency of the solution method.

for a solution of the time t+1 expected transition path is established.

Iteration on the above process Q times yields a Q-period artificial time series for our model economy from which estimates of the moments of the model's variables can be obtained. Following a standard RBC approach (eg. Prescott, 1986), we replicate this entire process $R = 1000$ times to generate a large number of such artificial time series and a large sample of estimated moments the averages of which are then compared to real economic data.

In practice we set $T = 200$ for the solution of each transition path. This is adequate to ensure that the model converges to its long run balanced growth path with a high degree of numerical accuracy[11]. We set Q=200 but truncate the first observations to eliminate any dependence of the simulated series on the τ starting values. This yields time-series of equal length to our actual data sample (187 observations)[12]. We perform R=1000 replications in all cases.

Appendix D. Q-Statistics

The Q statisitics reported are calculated as in Cogley and Nason (1995).

$$Q = (\hat{c} - c)' \hat{V}^{-1} (\hat{c} - c)$$

where $\hat{c}$ and c are 8×1 vectors containing the relevant autocorrelations or impulse-response function coefficients for the sample and the simulations respectively. The middle term is a covariance matrix calculated as,

$$\hat{V} = \{\frac{1}{R}\} \sum_{i}^{R} (c_i - c)(c_i - c)', \quad c = \{\frac{1}{R}\} \sum_{i=1}^{R} c_i.$$

where c_i is the relevant autocorrelation or impulse response function obtained from replication i. Q is distributed as a $\chi^2(8)$.

[11] Lower values would be adequate with lower persistence in the stochastic process.

[12] See Gregory and Smith (1991) for the statistical rational behind requiring equal sample sizes.