Policy-Induced Mean Reversion in the Real Interest Rate?

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Abstract

This paper utilizes tests for a unit root that have power against nonlinear alternatives to provide empirical evidence on the time series properties of the *ex-post* real interest rate in the G7 countries. We find that the unit root hypothesis can be rejected in the presence of a nonlinear alternative motivated by theoretical literature on optimal monetary policy rules. This represents a reversal of the results obtained using standard linear unit root and cointegration tests. Tests for linearity reject this hypothesis for Canada, France, Italy and Japan for which we estimate nonlinear models capturing the dynamics of the interest rate.

Keywords: Fisher Effect; Unit Roots; Self-Exciting Threshold Autoregression;

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1 Introduction

In this paper, we re-examine the empirical evidence on the Fisher effect by employing unit root tests (Bec, Ben Salem and Carrasco, 2004, and Bec, Guay and Guerre, 2004) that are designed to have more statistical power against nonlinear alternatives. The tests used in most of the previous empirical literature are based on linear autoregressions in which the null hypothesis of a unit root is tested against a linear stationary alternative. Such tests have been shown to have low power in distinguishing between the unit root model and a nonlinear but stationary alternative - see Pippenger and Goering (1993), Balke and Fomby (1997), Caner and Hansen (2001). The nonlinear alternative used in this paper is motivated by theoretical work on optimal monetary policy rules by Orphanides and Wilcox (2002) which suggests a threshold model which is characterized by three regimes for the *ex-post* real interest rate. Threshold models were originally proposed by Tong (1983), subsequently modified by Chan (1993) to the case of unknown thresholds and now provide a framework that allows for different degrees of mean reversion in a time series. The threshold representation for the *ex-post* real interest rate is consistent with the concept of optimal monetary policy rules.

It is assumed that the monetary authority implements policy through its influence on a short-term interest rate such as the federal funds rate. The optimal monetary policy rule suggests that modest deviations of inflation from its long-run target should not trigger policy action. Within a policy-inaction band, the real interest rate may exhibit weak or no mean reversion. On the contrary, incipient inflation prompts monetary policy action that re-enforces mean reversion in the real interest rate. Balke and Fomby (1997) studied mean reversion in the real exchange rate using a similar nonlinear framework. By allowing the real interest rate to exhibit different degrees of mean reversion over time, the nonlinear model provides a theoretically motivated structural approach to understanding real interest rate persistence.

The Fisher effect occurs when fully anticipated inflation has a unit effect on nominal interest rates; thus, leaving real interest rates unchanged. Theoretical support for the Fisherian conjecture is not universal. The proposition holds in models (e.g. Sidrauski, 1967) in which the real interest rate is determined by a relation like the modified golden rule and therefore does not depend on monetary variables. However, it is violated in models in which the Tobin effect applies - see Tobin (1965). In such models, an increase in the inflation rate leads to an increase in the opportunity cost of holding real money balances and a reallocation of wealth towards capital. With constant returns in the production technology, the marginal product of capital (the real interest rate) falls. Recent theoretical literature continues to be divided on this issue. For example, Cooley and Hansen (1989) support the Fisher effect while Drazen (1981), Weiss (1980), and Romer (1986) oppose it.[1]

It is clear that empirical evidence must be brought to bear on this issue in order to provide guidance for future theoretical modelling. Unfortunately, the voluminous empirical literature on this issue has not shown signs of convergence. While long-run money neutrality has received some empirical support, the evidence on long-run money superneutrality and the Fisher effect is mixed - see Bullard (1999).

Recent work on the Fisher effect has focused on the time series properties of the macrovariables. Nominal interest rates and inflation rates appear to be nonstationary variables - i.e. contain a unit root. However, a linear combination of these variables - such as their difference - may be stationary. In such case, the real interest rate will be mean reverting and the two variables will be cointegrated with a cointegrating vector $(1, -1)$. Mishkin (1992) used post-war US data and found that the nominal interest rate and inflation are cointegrated. Nonetheless, he points out that the results were sensitive to the choice of time period. Crowder and Hoffman (1996), using tax-adjusted US data, found strong support for a cointegrating relationship between nominal interest rates and inflation. Further, they were unable to reject the hypothesis that the cointegrating vector was $(1, -1)$; thus, supporting a one-to-one relationship between inflation and interest rates.

Alternative testing strategies for long-run neutrality propositions based on bivariate structural vector autoregressions were developed by Fisher and Seater (1993) and King and Watson (1997). The King and Watson tests have been applied to international data sets and provided mostly negative results for the Fisher effect and long-run money superneutrality - see Weber (1994), Koustas (1998), Koustas and Serletis (1999).

One possible explanation for the seemingly conflicting empirical results on the Fisher effect is that there may have been one or more structural breaks during the sample period. Ignoring structural breaks, biases the unit root tests in favour of accepting the null hypothesis of a unit root and against finding cointegration. Perron (1989) produced Monte Carlo evidence confirming this bias in the unit root tests. Garcia and Perron (1996), Caporale and Grier (2000) and Bai and Perron (2003) have tested for structural changes in the real interest rate using US data for the periods 1961-1986 and 1961-1992. They found evidence in favour of the Fisher effect once they allowed for occasional mean shifts in the real interest rate. However, Phillips (2005) points out that numerous mean shifts are required in order to characterize the data when longer samples on the real interest rate are used. One drawback of this approach is that it can lead to curve fitting with ex-post rationalization of

[1] Orphanides and Solow (1990) provide an in-depth review of the earlier inflation-and-growth literature.

the regime shifts. Phillips (2005) adopted instead a nonparametric approach based on modelling the spatial densities of the real interest rate and found that the US real interest rate is fractionally nonstationary hence allowing for the real rate to be stationary with long memory.

We propose a different approach to study data on the real interest rate based on nonlinear models. The main motivation for allowing a nonlinear structure is twofold. First, unit root tests based on nonlinear alternatives have been shown to have better finite sample properties and second, the nonlinear models used are theoretically motivated, flexible, and parsimonious.

We obtain rejections of the unit root hypothesis in the ex-post real interest rate for all of the G7 countries, except for the UK, when we use a Self-Exciting Threshold Autoregression (SETAR) model as the alternative. This essentially reverses the results from linear unit root and cointegration tests. Where our tests reject linearity we estimate SETAR models and test other hypotheses of interest (such as the commonly used Band-TAR model). We find that the real interest rate is mean reverting in the outer regimes where monetary policy is active. When inflation is high enough, in absolute value, it triggers an increase in the nominal interest rate to push the real interest rate toward its long-run level. On the other hand, when inflation is within a band, monetary policy is inactive and this weakens mean reversion in the real interest rate. Kapetanios, Shin and Snell (2003) also studied real interest rate dynamics as a nonlinear process using a smooth transition autoregressive process. Earlier work on the interest rate by González and Gonzalo (1997) focused on an atheoretical two regime threshold unit root model that allows for mean reversion in one regime and a random walk in another. Their results are similar to ours but our approach differs in that our empirical models under the nonlinear hypothesis match a theoretical model that is characterized by three regimes and a possible inaction band. Overall, our empirical findings are encouraging for the Fisher effect.

The next Section summarizes standard linear tests for the Fisher effect and outlines the proposed nonlinear alternative. Section 3 discusses the econometric methodology used to obtain the empirical results that are presented in Section 4. Finally, Section 5 closes with a brief summary and conclusion.

2 The Fisher Effect

2.1 The Linear Framework

Irving Fisher (1930) introduced the idea of the *ex-ante* real interest rate - nominal interest rate minus expected inflation - as the compensation of lenders based on their beliefs about inflation. If i_t^m denotes the m-period nominal interest rate at time t, $\pi_t^{e,m}$ the expected rate of inflation from time t to time $t+m$, and $r_t^{e,m}$ the corresponding *ex-ante* real interest rate, then the Fisher equation is

$$1 + i_t^m = (1 + r_t^{e,m})(1 + \pi_t^{e,m}) \tag{1}$$

For low rates of inflation, the Fisher equation can be approximated by

$$i_t^m = r_t^{e,m} + \pi_t^{e,m} \tag{2}$$

where both $r_t^{e,m}$ and $\pi_t^{e,m}$ are unobservable. Defining π_t^m as the m-period inflation rate, we obtain the following relation between the *ex-post* (r_t^m) and the *ex-ante* real interest rate:

$$r_t^m = i_t^m - \pi_t^m = r_t^{e,m} + (\pi_t^{e,m} - \pi_t^m) = r_t^{e,m} + \epsilon_t \tag{3}$$

where, the inflation forecast error (ϵ_t) has a zero mean, is uncorrelated over time and orthogonal to any other economic variables (assuming rational expectations).

Equation (3) suggests that the gap between the *ex-post* and the *ex-ante* real interest rate is stationary. Consequently, if it is found that the *ex-post* real interest rate is stationary, one may conclude that the *ex-ante* real interest rate is also stationary and the Fisher effect holds.

In the context of cointegration, equation (2) suggests that if the *ex-ante* real interest rate is stationary with mean α, the coefficient β should be unity in a regression of the form

$$i_t^m = \alpha + \beta\pi_t^{e,m} + \eta_t \tag{4}$$

and the residuals (η_t) should be stationary. The *ex-post* real interest rate could then be written as

$$r_t^m = i_t^m - \pi_t^m = \alpha + (\pi_t^{e,m} - \pi_t^m) + \eta_t = \alpha + \epsilon_t + \eta_t \tag{5}$$

which implies stationary fluctuations around a constant level α. Thus, in the linear context, real interest rates may deviate from their mean only temporarily as a result of uncorrelated inflation forecast errors or stationary shocks when the Fisher effect holds.

2.2 A Nonlinear Framework

This subsection draws heavily from Orphanides and Wilcox (2002) who postulate a conventional macromodel consisting of an aggregate demand relationship and an expectations-augmented Phillips curve. Their model is summarized as follows:

$$y_t = \rho y_{t-1} - \sigma(r_t - r^*) + u_t \tag{6}$$

$$\pi_t = \pi_t^e + \alpha y_t + e_t \tag{7}$$

where y_t is the deviation of output from potential (measured in logarithms) and the parameter ρ is a positive fraction capturing the persistence of the output gap. The deviation of the real interest rate from its steady-state level is denoted by $r_t - r^*$, σ is a positive parameter, and u_t is an aggregate demand shock. Further, π_t and π_t^e are inflation and expected inflation respectively, α is a positive parameter, and e_t is an aggregate supply shock.

The policy maker is assumed to suffer loss from deviations of inflation from its intermediate target $(\pi_t - \widetilde{\pi})$ and deviations of output from potential according to the following loss function:

$$\mathcal{L} = (\pi_t - \widetilde{\pi})^2 + \gamma y_t^2 + \psi\,|y_t| \tag{8}$$

where $\gamma \geq 0$, and $\psi \geq 0$. It is further assumed that the intermediate target for inflation is a positive fraction λ of the lagged inflation rate (i.e. $\widetilde{\pi} = \lambda \pi_{t-1}$). The lower the value of λ, the more aggressively the intermediate inflation target is adjusted to its long-run value of zero. It is shown that if neither the demand nor the supply shock can be anticipated, and inflation expectations are static ($\pi_t^e = \pi_{t-1}$) the optimal rule for the policy maker is opportunistic regarding its response to inflation:

$$i_t - \pi_{t-1} = r^* + \frac{\rho}{\sigma} y_{t-1} + \begin{cases} \frac{\alpha(1-\lambda)}{\sigma(\alpha^2+\gamma)}(\pi_{t-1} - \overline{\pi}_{t-1}) & \text{if } \pi_{t-1} > \overline{\pi}_{t-1} \\ & \text{if } \underline{\pi}_{t-1} \leq \pi_{t-1} \leq \overline{\pi}_{t-1} \\ \frac{\alpha(1-\lambda)}{\sigma(\alpha^2+\gamma)}(\pi_{t-1} - \underline{\pi}_{t-1}) & \text{if } \pi_{t-1} < \underline{\pi}_{t-1} \end{cases} \tag{9}$$

where $\overline{\pi}_{t-1} = \frac{\psi}{2\alpha(1-\lambda)}$ and $\underline{\pi}_{t-1} = -\frac{\psi}{2\alpha(1-\lambda)}$ are an upper and a lower threshold respectively.

The opportunistic monetary policy rule suggests that as long as inflation is within the band defined by the lower and upper threshold $\left[\underline{\pi}_{t-1}, \overline{\pi}_{t-1}\right]$, it does not warrant a policy response. Once inflation exceeds the upper threshold, it triggers an increase in the nominal interest rate intended to bring the real interest rate up toward its long-run level. An analogous argument can be made for rates of inflation that are below the lower threshold. From an empirical standpoint, this model suggests that optimal monetary policy reinforces the mean-reverting properties of the *ex-post* real interest rate when lagged inflation lies outside a certain band. When lagged inflation - or equivalently the *ex-post* real interest rate - lies within the policy inaction band, it does not trigger a policy response and mean reversion is generally weaker.

The width of the policy-inaction band is determined by the values of the parameters entering the expressions for the thresholds. For example, a monetary authority with a gradualist approach to disinflation (high λ) will have a relatively wide band of policy inaction. The same conclusion holds for a monetary authority that faces a flat expectations-augmented Phillips curve (low α) and/or experiences high loss from deviations of output from potential (high ψ).

3 The Econometric Methodology

Linear unit root tests on the demeaned *ex-post* real interest rate (z_t) are based on the following auxiliary regression:

$$\Delta z_t = \mu + \rho z_{t-1} + \sum_{i=1}^{p} \alpha_i \Delta z_{t-i} + \epsilon_t \tag{10}$$

where μ is a drift term and ϵ_t is a white noise disturbance. The null hypothesis of a unit root is tested by imposing the restriction $\rho = 0$ against the alternative $\rho < 0$ and testing it using the Dickey and Fuller (1981) τ_μ or τ_τ statistics. If the null cannot be rejected using appropriate critical values, it is concluded that deviations of z_t from its mean are infinitely persistent.

In the context of the optimal monetary policy rule described in the previous section, real interest rate deviations from its mean will be more persistent - ρ will be closer to zero - inside the policy-inaction band. When real interest rates move outside this band, they trigger monetary policy action that strengthens mean reversion (ρ becomes more negative in the outer regimes). A linear specification like (10) amounts to pooling observations from three different regimes and leads to biased estimates of ρ.

To deal with this issue, we use the following very general 3-regime SETAR model:

$$\Delta z_t = \epsilon_t + \begin{cases} \mu_1 + \rho_1 z_{t-1} + \sum\limits_{i=1}^{p} \alpha_{1i} \Delta z_{t-i} & \text{if } z_{t-d} \leq -\theta \\ \mu_2 + \rho_2 z_{t-1} + \sum\limits_{i=1}^{p} \alpha_{2i} \Delta z_{t-i} & \text{if } -\theta < z_{t-d} < \theta \\ \mu_3 + \rho_3 z_{t-1} + \sum\limits_{i=1}^{p} \alpha_{3i} \Delta z_{t-i} & \text{if } z_{t-d} \geq \theta \end{cases} \tag{11}$$

In (11), ϵ_t is a white noise disturbance common across regimes and the dynamics of the process are captured by Δz_{t-i}. The transition variable is z_{t-d} with a delay, d, that is unknown and, in theory, that could be estimated. However, as reported by Bec, Ben Salem and Carrasco (2004), the estimate of d is not precise. For this reason, and following others, we set $d = 1$. The threshold, θ, is assumed to be symmetric around zero but unknown. Within the band $[-\theta, \ \theta]$, real interest rate deviations from its mean may be infinitely persistent ($\rho_2 = 0$) because the monetary authority tolerates them. However, ρ_2 will be negative if the Fisher effect holds. In the outer regimes, ρ_i will be negative as a result of monetary policy action even if the Fisher effect is invalid. This model, therefore, allows for convergence to $-\mu_i/\rho_i$ (an equilibrium point) outside and inside the band as well as non-convergence inside the band. Constrained versions of this 3-regime SETAR model include the symmetric mirroring TAR (SMTAR) model which imposes a common convergence point and common convergence speed (see Bec, Guay and Guerre (2004)) as well as the Band TAR (BTAR) model which further imposes convergence to the edges of the band (see Balke and Fomby (1997), Obstfeld and Taylor (1997) and Taylor (2001)). Other restrictions could be imposed such as a unit root in the middle regime and zero drift in the outside regimes (see Kapetanios and Shin (2002)). Bec, Ben Salem, and Carrasco (2004) have shown that a process like (11) will be globally stationary - under certain conditions - even if it has a unit root in the middle regime. The same authors propose a test of the unit root hypothesis that is specifically designed to have increased statistical power when the alternative hypothesis is a SETAR model like (11). The distribution of the test statistic was derived under the hypothesis $\rho_1 = \rho_2 = \rho_3 = 0 = \mu_1 = \mu_2 = \mu_3$ and $\alpha_{1i} = \alpha_{2i} = \alpha_{3i}$ for $i = 1, 2, \ldots, p$ but the test is constructed for $\rho_1 = \rho_2 = \rho_3 = 0$ to ensure that only nonstationarity is tested.

The following steps describe the estimation and testing procedures:

1. The data on z_t (in absolute value) is sorted in ascending order. The data is demeaned prior to the analysis since we assumed that the threshold model is symmetric around zero which will yield a threshold for the real interest rate of the form $\pm\theta$. For the unit root test to be free of nuisance parameters, the threshold is contained in an interval $\left[\underline{\theta}, \overline{\theta}\right]$ such that $\beta\%$ of the $|z|$ will lie below or above the lower and upper bounds, $\underline{\theta}$ and $\overline{\theta}$. According to common practice we set $\beta = 15$. This leads to at least 30% of the sorted observations falling outside and inside the middle regime. This way, the estimated SETAR model is not unduly influenced by a few important outliers.

2. Conditional on the estimated optimal threshold, we compute the following Likelihood Ratio test statistic:
$$LR_T(\theta) = T \ln\left(\frac{\widetilde{\sigma}^2}{\widehat{\sigma}^2}\right) \tag{12}$$
where $\widetilde{\sigma}^2$ is the restricted estimate of the variance (imposing $\rho_1 = \rho_2 = \rho_3 = 0$ in (11)) and $\widehat{\sigma}^2$ is the unrestricted estimate. Since the test is based on an estimated value of θ we use the supremum test defined as
$$\sup LR \equiv \sup_{\theta\in\left[\underline{\theta}, \overline{\theta}\right]} LR_T(\theta) \tag{13}$$

3. The estimate of the $\sup LR$ is compared to its critical value. If the test statistic exceeds the simulated empirical critical value, the unit root hypothesis is rejected. Although some critical values for the $\sup LR$ appear in Bec, Ben Salem and Carrasco (2004) we obtained empirical critical values by simulating model (10) under the null hypothesis of a unit root
$$\Delta z_t = \mu + \sum_{i=1}^{p} \alpha_i \Delta z_{t-i} + \epsilon_t \tag{14}$$
where $\epsilon_t \sim N(0, \sigma^2)$. Model (14) was estimated for the G7 economies and the parameters μ, α_1, α_2, and σ were set at the average over all countries with $p = 0, 2$. This yielded the following parameter values for the simulations: $\mu = 0$, $\sigma = 0.03$ for $p = 0$ and $\mu = 0$, $\alpha_1 = -0.44$, $\alpha_2 = -0.27$, and $\sigma = 0.027$ for $p = 2$. The sample size for the simulations was set to 200 to reflect the available sample size and the number of replications was set to $10,000$. The empirical critical values are reported in Table 4.

4. For countries that reject the unit root null, the analysis proceeds with $\sup Wald$ tests for linearity suggested by Hansen (1996, 1997). We test the null hypothesis of linearity by imposing the following restrictions $\alpha_{1i} = \alpha_{2i} = \alpha_{3i}$ for $i = 1, 2, \ldots, p$, $\mu_1 = \mu_2 = \mu_3$ and $\rho_1 = \rho_2 = \rho_3$. Since the distribution of this test under the null depends on nuisance parameters, we simulate (using 1000 replications) the p-values for the heteroskedasticity-robust $Wald$ test, $Wald_T(\theta)$. Again, since the threshold is unknown we use the supremum version of the $Wald$ test
$$\sup Wald \equiv \sup_{\theta \in \left[\underline{\theta},\, \overline{\theta}\right]} Wald_T(\theta) \tag{15}$$

5. Specification tests for these countries are conducted following a general-to-specific approach to determine the appropriate nonlinear model for the *ex-post* real interest rate. These tests are conditional on the optimal threshold that is selected by conducting a grid search for θ in the interval $\left[\underline{\theta},\, \overline{\theta}\right]$. Equation (11) is repeatedly estimated by nonlinear least squares for different values of the threshold and the value that minimizes the sum of squared residuals is selected.

4 Estimation Results

The data set used in the estimation consists of the 3-month Treasury Bill rate (or other short-term interest rate in a few cases) and the Consumer

Price Index for the G7 countries. The data was converted to quarterly from a monthly frequency through averaging. The annual rate of inflation was computed as the log-difference of the quarterly CPI times 400. Finally, the rate of inflation was aligned with the appropriate quarterly observation of the interest rate. The sample spans the period 1960 to 2004 in most cases. The sample for Germany starts in 1962 and the samples for France and the UK start in 1970. The US data were obtained from the Federal Reserve on-line data base FRED. The rest of the data were obtained from Datastream.[2]

Table 1 presents some standard unit root and cointegration tests for comparability with previous work. The results from the Augmented Dickey Fuller (ADF) test for a unit root are reported in columns 2 and 5. The auxiliary autoregressions include a constant and have a lag length that was selected using the BIC. The p-values for the test - reported in parentheses - indicate the probability of rejecting the unit root null when it is in fact true. In all cases, the p-values are greater than the 5 percent significance level rejecting stationarity for both the nominal interest rate and inflation. These results were confirmed by the Generalized Least Squares version of the ADF test (ADF-GLS) suggested by Elliot et al. (1996). This test increases power by obtaining estimates of the parameters for the deterministic regressors and using them to detrend the data prior to the estimation of the ADF auxiliary autoregression. In all cases, the reported t-statistics were less (in absolute terms) than the 5 percent critical value thus rejecting stationarity.

Kwiatkowski et al. (1992) argue that the way in which classical hypothesis testing is carried out ensures that the null hypothesis is accepted unless there is strong evidence against it. They have proposed tests (known as the KPSS tests) of the hypothesis of stationarity against the alternative of the unit root. The p-values for the KPSS tests - reported in columns 4 and 7 - are less than 5 percent in all cases except the German interest rate. Thus, with the possible exception of the German interest rate, the null hypothesis of stationarity can be rejected.

Having found that nominal interest rates and inflation rates are probably integrated variables, $I(1)$, we test for cointegration by checking the residuals from a regression of the nominal interest rate on inflation for a unit root. The ADF tests on the residuals and their associated p-values are reported in column 8. In all cases the p-values exceed the 5 percent significance level, thus, rejecting stationarity in the residuals (rejecting cointegration). We also tested for the full Fisher effect by restricting the cointegrating vector to be $(1, -1)$. This is equivalent to testing for a unit root in the ex-post real interest rate. The ADF tests, reported in column 9, fail to reject the unit

[2] The data and computer code used in this paper are available from the authors upon requests.

root (i. e. reject cointegration) in all cases. It should be noted that inability to reject the unit root null in the presence of a linear stationary alternative is very pronounced - the minimum p-value is 15 percent and the maximum is 47 percent.

In Table 2, we report the results from unit root tests on the ex-post real interest rate that are designed to have increased power against nonlinear alternatives. The nonlinear alternative hypothesis is the SETAR model described by equation (11). This model is more general than restricted versions considered by Kapetanios and Shin (2002) who impose a unit root in the middle regime, $\rho_2 = 0$, and zero drift in all regimes, $\mu_i = 0$ for $i = 1, 2, 3$. Further, the model encompasses the SMTAR and BTAR models that have been studied by a number of authors (see above). The former is a special case of (11) when $\mu_1 = -\mu_3$, $\rho_1 = \rho_3$ and $\alpha_{1i} = \alpha_{2i} = \alpha_{3i}$ for $i = 1, 2, \cdots, p$ while the latter impose further restrictions on $\mu_{1.}$ The number of lagged differences in (11) was set according to the BIC and is reported in column 2.

The results of the $\sup LR$ test statistics, reported in column 3, indicate that the null hypothesis of nonstationarity is rejected for all countries except for the UK (also found by Kapetanios, Shin and Snell (2003)). In particular, the statistics exceed their 1-percent simulated critical values for Canada, Germany, Italy and Japan, the 5-percent critical value for France and the 10-percent critical value for the US. We conclude that for 6 out of the 7 countries the unit root in the ex-post real interest rate can be rejected in favour of the nonlinear alternative represented by (11). This represents almost a complete reversal of the results obtained under the assumption of linearity. These results are not surprising as the ADF tests for a unit root have been shown to have low power against nonlinear stationary alternatives.

Having rejected the unit root null in the ex-post real interest rate, we proceed with tests for linearity. The $\sup Wald$ tests, reported in column 4 of Table 2, test the null hypothesis of linearity against model (11) by imposing the following restrictions: $\mu_1 = \mu_2 = \mu_3$, $\rho_1 = \rho_2 = \rho_3$, and $\alpha_{1i} = \alpha_{2i} = \alpha_{3i}$ for $i = 1, 2, \cdots, p$. We report the p-values computed using both the homoskedastic $Wald$ and heteroskedastic $Wald$ tests but focus on the heteroskedasticity-robust test since the variances (not reported) differ substantially across regimes. The results of column 4 suggest that linearity can be very strongly rejected in all cases except for Germany and the US which are dropped from further analysis.

For the four countries that rejected the linearity restrictions, the real interest rate is modelled by (11). The estimated thresholds for the same countries are reported in column 5 of Table 2 and range from [$\pm$0.94%] (Canada) to [$\pm$5.21%] (Japan), France has a threshold of [$\pm$4.01%] while the threshold for Italy is [$\pm$4.43%]. For illustration purposes, we computed the inflation rate thresholds associated with the reported interest rate thresholds. These range from [±2.1%] (Canada) to [±13.68%] (Italy) while they are [±12.05%] for France and [±10.4%] for Japan. Because these are preliminary estimates we delay their discussion until after the specification tests.

The last four columns in Table 2 report tests for a number of restrictions on (11). We find that the nonstationary middle regime hypothesis, $\rho_2 = 0$ (last column of Table 2), cannot be rejected for France and Italy but it is rejected for Canada and Japan. This finding suggests that the full Fisher effect is valid inside the middle regime in the case of Canada and Japan. In the case of France and Italy, the real interest rate is globally stationary but locally nonstationary.

Kapetanios and Shin (2002) proposed a version of (11) that imposes the restriction of zero drift in all regimes. Wald tests for the restriction $\mu_1 = \mu_2 = \mu_3 = 0$, reported in column 9 of Table 2, reject it for significance levels greater than 11% for all countries except Japan. Estimating a constrained model could have, therefore, introduced bias in the unit root testing procedure. The SMTAR hypothesis imposes the restriction $\mu_1 = -\mu_3$, $\rho_1 = \rho_3$. Wald tests for this restriction, reported in column 10 of Table 2, are unable to reject the SMTAR hypothesis for Canada, France and Japan for significance levels smaller than 11%. The SMTAR hypothesis is rejected for Italy suggesting asymmetric mean reversion in the real interest rate outside the policy-inaction band. Finally, the hypothesis of common dynamics across regimes, $\alpha_{1i} = \alpha_{2i} = \alpha_{3i}$ for $i = 1, 2, \cdots, p$ (column 11) cannot be rejected for Japan.

Our results can be summarized as follows:

1. The *ex-post* real interest rate is stationary (allowing for nonlinear adjustment) for all countries in the sample except the UK.

2. The *ex-post* real interest rate can suitably be modeled as a general 3-regime SETAR model, for Canada, France, Italy, and Japan.

3. The *ex-post* real interest rate in Canada and France follows a SMTAR model (or the BTAR as a special case) with common speed of adjustment in the outside regimes and common (but opposite in sign) convergence points.

4. The nonlinear model for the *ex-post* real interest rate in Japan strongly supports common dynamics in the three regimes.

We now proceed to the versions of the restricted SETAR models that were suggested by our previous tests. The results are reported in Table 3. Because we re-estimate models that are more restrictive we test for the unit root hypothesis again, allowing for a nonlinear alternative. A rejection of the null hypothesis will provide further evidence in favor of the Fisher effect. In contrast, Kapetanios and Shin (2002) and Obstfeld and Taylor (2001) start with a restricted model which may lead to making a type II error. We apply the adaptative unit root sup $Wald$ test for $\rho_1 = \rho_2 = \rho_3 = 0$ proposed by Bec, Guay and Guerre (2004) for the countries that did not reject the SMTAR hypothesis. The test is similar to the one used in this paper but specifies a threshold set that can adapt to the hypothesis that is being tested. In particular, the threshold set is made wider under the stationary (but nonlinear) alternative than under the null hypothesis of a unit root. The test proposed by Bec, Guay and Guerre (2004) covers a wide range of applications including the BTAR model. The results of the adaptative unit root test (the statistics for the unbounded and bounded thresholds are the same) are reported in column 2 of Table 3. Because the BTAR specification imposes a number of restrictions on (11) we used the BIC to find the optimal number of (common) lagged differences, 1 for France and 3 for Canada. The reported test statistics are greater than the 10% critical values for France and greater than the 5% critical values for Canada providing further evidence against the unit root hypothesis but this time in favor of a very restricted SMTAR model. For this reason we proceed with the estimation of the BTAR which is characterized by mean reversion of the outside regimes toward the edge of the inaction band:

$$\Delta z_t = \sum_{i=1}^{p} \alpha_{*i} \Delta z_{t-i} + \epsilon_t + \begin{cases} \rho_1 (z_{t-1} + \theta) & \text{if } z_{t-1} \leq -\theta \\ \rho_2 z_{t-1} & \text{if } -\theta < z_{t-1} < \theta \\ \rho_1 (z_{t-1} - \theta) & \text{if } z_{t-1} \geq \theta \end{cases} \qquad (16)$$

The parameter estimates are all significant with the exception of the autoregressive parameters in the middle regime indicating the presence of a unit root in the middle band and evidence that a full Fisher effect does not hold in the middle regime for Canada and France. Mean reversion towards the edges of the band, [$\pm$4.15%] for Canada and [$\pm$5.22%] for France, occurs at a slow, but similar, speed. The autoregressive coefficients are 0.85 for Canada and 0.82 for France corresponding to deviations with a half-life of about $\ln(0.5)/\ln(0.85) = 4.26$ quarters for Canada and of about $\ln(0.5)/\ln(0.82) = 3.49$ quarters for France (see Taylor (2001) for an in-depth analysis). The middle regime is characterized by a band of [$\pm$2.29%] for Canada and [$\pm$7.57%] for France suggesting that France tolerates a much higher inflation rate before an increase in the nominal interest rate will bring the real interest rate up toward its lower threshold. In Figure 1 we see that the lower and upper regimes for the real interest rates in Canada and France are characterized by episodes of high inflation during the 1970s and its rapid deceleration in the 1980s.

The general 3-regime SETAR model was estimated for Italy and the results indicate that the unit root hypothesis is plausible in the middle regime providing more evidence against the Fisher effect over a range of the real interest rate. The autoregressive coefficient of the lower regime is very similar to the one for France and reveals a slow mean reversion with a half-life of $\ln(0.5)/\ln(0.81) = 3.29$ quarters. We can see in Figure 2 that Italy is characterized by periods of high volatility in the real interest rate which correspond to very high and volatile inflation during the late 1970s. For this reason the inflation rate equivalent threshold is quite high at [$\pm 13.68\%$]. These results should, however, be interpreted with caution since a unit root could also be present in the upper regime. Lastly, a common dynamics 3-regime SETAR was estimated for Japan. We find that Fisher effect does not hold in the middle regime (unit root is present) but holds otherwise. The autoregressive coefficients are significant and imply a very fast mean reversion to an equilibrium point for the real interest rate of 2.27 for the upper regime and a reversion towards 0 for the lower regime (since the constant in the lower regime is not significant). The inflation rate threshold is quite low at [$\pm 2.9\%$] and very similar to the one in Canada. It is interesting to note that from Figure 2 we see that Japan had a very high inflation in 1974 implying a very low real interest rate. This persisted for a few years and could, in part, drive the results found here.

5 Concluding Remarks

Standard tests for a unit root, in which the alternative hypothesis is a stationary linear model, reveal that the *ex-post* real interest rate of the G7 economies is not mean reverting. This implies that the nominal interest rate and inflation can drift apart from one another indefinitely thus invalidating the Fisher effect. However, it is well documented that linear unit root and cointegration tests lack power against nonlinear alternatives.

In this paper, we have used recent methodological advances to test the unit root hypothesis against a very general nonlinear alternative. Our use of a three-regime SETAR model as the alternative hypothesis is motivated by recent theoretical literature suggesting different degrees of mean reversion in the real interest rate, depending on whether monetary policy is actively engaged or in a stand-by mode. unit root tests of the type used in the paper have been shown to have increased power against nonlinear alternatives. This was confirmed by our empirical results which suggest that, overall, the unit root in the real interest rate can be rejected in favour of a SETAR model. Further, our linearity tests reject this hypothesis for Canada, France, Italy, and Japan. The three-regime SETAR models, which encompass the commonly used Band-TAR model, were estimated for these countries and seem to capture the dynamics of the real interest rate well.

We found that the *ex-post* real interest rate is globally stationary in all the G7 countries except in the UK. In most cases, the real interest rate is mean reverting in the outer regimes where monetary policy is active and we characterized mean reversion in terms of inflation. When inflation is high enough, in absolute value, it triggers an increase in the nominal interest rate to push the real interest rate toward its long-run level. On the other hand, when inflation is within a band, we find that for Canada, France, Italy and Japan no policy response is required. We also find that the middle regime is very tight for Canada and Japan while it is much wider for France and Italy, probably reflecting differences in the degree of policy intervention. Overall, our empirical findings are encouraging for the Fisher effect as a global proposition, while revealing significant departures locally. Moreover, they indicate that often mean reversion in the real interest rate may be the result of active monetary policy rather than a bond market outcome as conjectured by Irving Fisher. The empirical evidence presented seems to be unfavourable to theoretical models - particularly short-run models - that have an embedded Fisher relationship assumed to hold at all times.

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Table 1. Unit root and cointegration tests against linear alternatives

	i_t			π_t			Cointegration Tests	
Country	ADF	$ADF-GLS$	$KPSS$	ADF	$ADF-GLS$	$KPSS$	$i_t = \alpha + \beta\pi_t + e_t$	$i_t - \pi_t = \alpha + e_t$
Canada	−1.78	−1.51	0.59	−2.13	−1.44	0.68	−2.11	−2.30
	(0.46)		(0.02)	(0.27)		(0.01)	(0.27)	(0.20)
France	−1.05	−1.48	1.40	−1.44	−1.50	1.99	−1.45	−2.24
	(0.68)		(0.00)	(0.51)		(0.00)	(0.61)	(0.25)
Germany	−3.04	−1.69	0.38	−2.80	−2.24	0.71	−2.39	−2.24
	(0.06)		(0.08)	(0.07)		(0.01)	(0.19)	(0.24)
Italy	−1.33	−1.05	0.73	−1.48	−1.18	0.68	−1.52	−2.24
	(0.63)		(0.00)	(0.60)		(0.01)	(0.56)	(0.24)
Japan	−1.14	−0.36	2.17	−1.72	−1.67	1.49	−1.47	−2.62
	(0.61)		(0.00)	(0.43)		(0.00)	(0.50)	(0.15)
UK	−1.26	−2.06	1.12	−1.69	−1.54	1.70	−1.67	−1.80
	(0.62)		(0.00)	(0.49)		(0.00)	(0.51)	(0.47)
US	−2.02	−1.69	0.52	2.70	−2.24	0.51	−2.14	−2.37
	(0.31)		(0.03)	(0.10)		(0.04)	(0.27)	(0.18)

Notes: For the ADF and KPSS tests, values in parentheses are bootstrap p-values (see Park 2003). The null hypothesis for the ADF tests is the presence of a unit root. The null hypothesis for the KPSS tests is of level stationarity. The 5% critical value for the ADF-GLS test is -2.93. For the cointegration tests the null is of a unit root in the residuals implying no cointegration. The number of lagged differences used for the tests was found using the BIC. Sample periods are defined in section 4.

Table 2. Unit root and linearity tests on the real interest rate against a SETAR alternative

								Wald tests for			t-test for
	Lagged				% of obs. in			Zero		Common	Unit Root in
Country	Differences (p)	$\sup LR$	$\sup Wald$	Threshold	Lower	Middle	Upper	Drift	SMTAR	Dynamics	Middle Regime
Canada	0	26.99	16.48/11.14	$\pm 0.94/\pm 2.10$	32	29	39	5.99	5.99		−3.28
			(0.00/0.01)					(0.11)	(0.11)		(0.00)
France	0	17.72	7.74/65.30	$\pm 4.01/\pm 12.05$	11	79	10	44.12	1.85		−1.15
			(0.25/0.00)					(0.00)	(0.39)		(0.25)
Germany	0	32.00	2.65/3.04								
			(0.33/0.66)								
Italy	0	40.49	26.34/13.027	$\pm 4.43/\pm 13.68$	13	70	17	6.57	5.24		0.30
			(0.00/0.00)					(0.09)	(0.07)		(0.76)
Japan	4	22.71	29.99/23.73	$\pm 5.21/\pm 10.41$	10	80	10	1.10	14.08	8.62	−2.94
			(0.18/0.01)					(0.78)	(0.17)	(0.36)	(0.00)
UK	4	9.37									
US	0	16.47	5.00/8.58								
			(0.30/0.20)								

Notes: The $\sup LR$ test considers the unit root null. Its simulated critical values are reported in Table 4. The $\sup Wald$ test considers the linear null. The numbers in parentheses of column 4 are simulated asymptotic p-values for the homoskedastic and heteroskedastic $Wald$ tests. The threshold variable, z_{t-1}, is the lagged real interest rate. Numbers in parentheses in the last four columns are standard p-values. The second entry in the threshold column is the inflation rate associated for a given interest rate threshold. Sample periods are defined in section 4.

Table 3. Estimates of threshold models

BTAR	Adaptive												% of obs. in	
	sup $Wald$				α_{*1}	α_{*2}	α_{*3}	ρ_1	ρ_2	$\rho_3 = \rho_1$	Threshold	Lower	Middle	Upper
Country														
Canada	15.41				−0.29	−0.35	−0.23	−0.15	0.01	−0.15	±4.15/ ± 2.29	13	78	9
					(0.09)	(0.08)	(0.07)	(0.05)	(0.09)	(0.05)				
France	13.89				−0.19			−0.18	0.00	−0.18	±5.22/ ± 7.57	5	90	5
					(0.09)			(0.05)	(0.06)	(0.05)				
TAR													% of obs. in	
		μ_1	μ_2	μ_3	α_{*1}	α_{*2}	α_{*3}	ρ_1	ρ_2	ρ_3	Threshold	Lower	Middle	Upper
Country														
Italy		−8.22	0.04	−0.53				−1.17	0.02	−0.19	±4.43/ ± 13.68	13	70	17
		(3.23)	(0.20)	(2.16)				(0.39)	(0.08)	(0.29)				
Japan		0.39	−0.08	2.45	−0.41	−0.17	0.00	−0.99	−0.37	−1.09	±1.65/ ± 2.90	27	40	33
		(0.37)	(0.89)	(1.40)	(0.15)	(0.16)	(0.13)	(0.48)	(0.23)	(0.33)				

Notes: Robust standard errors are reported in parentheses. The 1, 5 and 10 percent critical values of the Adaptive sup $Wald$ tests are 19.33,14.84 and 12.99.for the unbounded case and 18.05, 13.83 and 11.98 for the bounded case. Sample periods are defined in section 4. The threshold variable, z_{t-1}, is the lagged real interest rate. The second entry in the threshold column is the inflation rate associated for a given interest rate threshold. The estimates for α_{*i} are for the common dynamics terms.

Table 4. Empirical critical values for $\sup LR$

	Quantiles	85%	90%	95%	99%
$p = 0$		14.249	15.558	17.655	22.384
$p = 2$		14.705	16.018	18.277	22.567

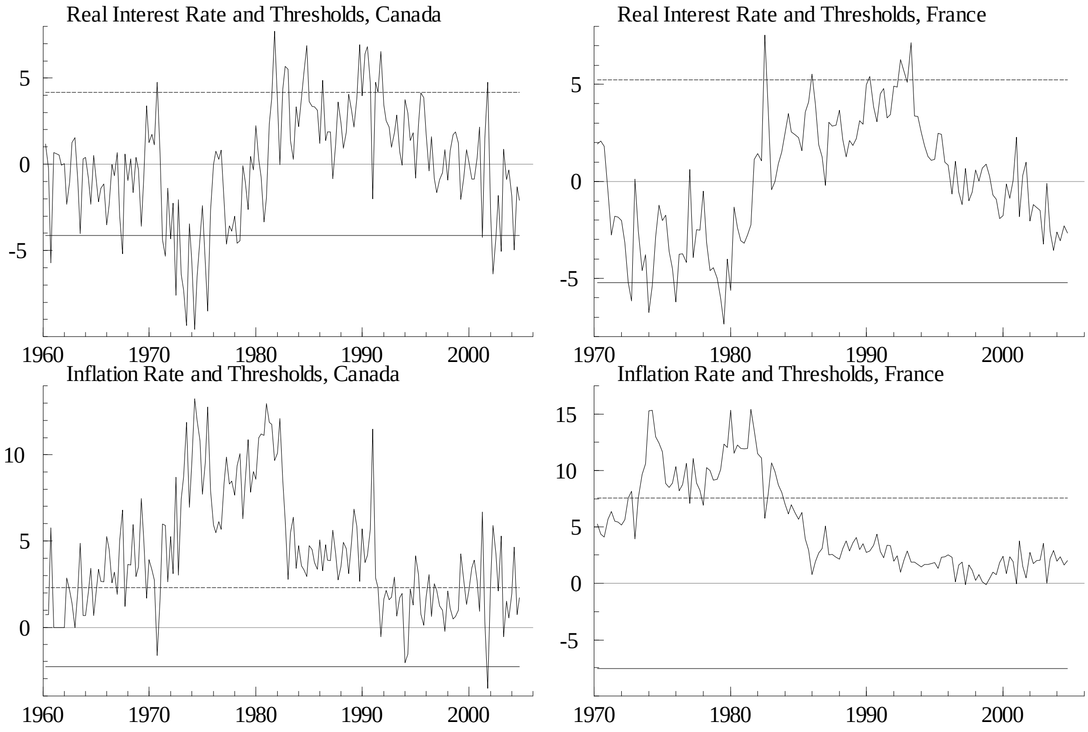

Figure 1: Canada and France

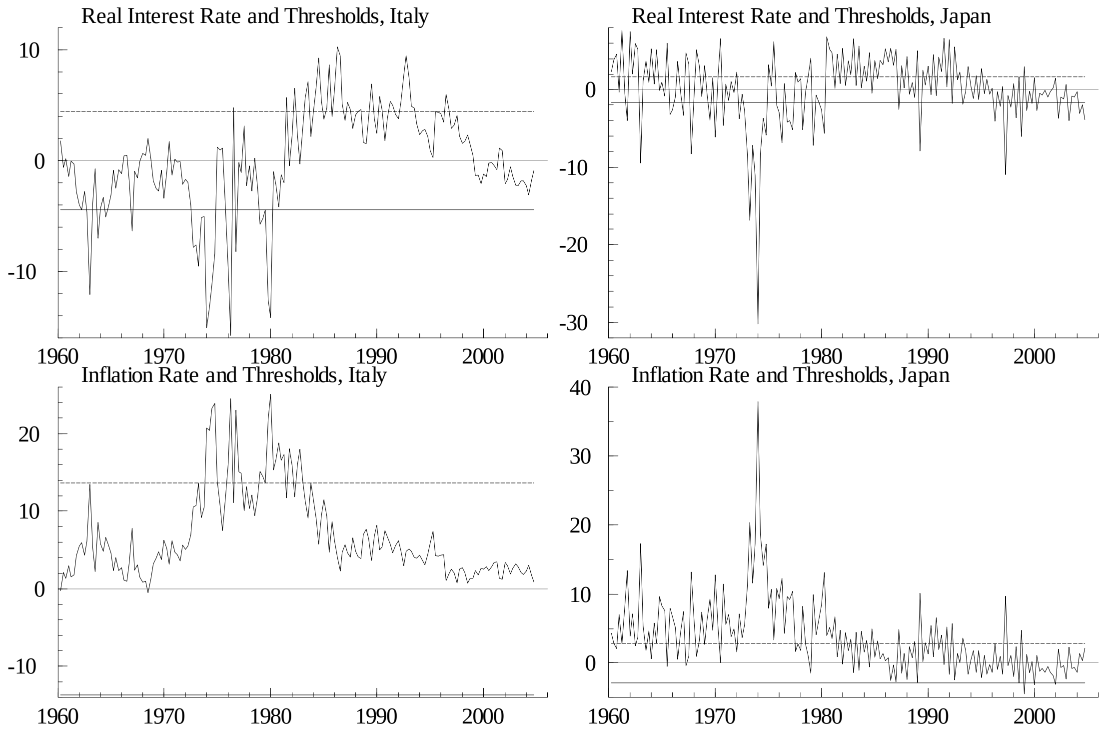

Figure 2: Italy and Japan