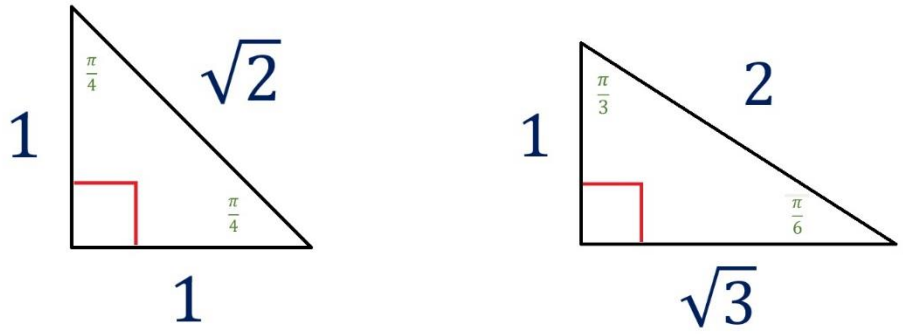
Trigonometric Angles Using Special Triangles

Besides the Unit Circle Method, we can use special triangles to evaluate certain trigonometric ratios using SOHCAHTOA.

There are two triangles to memorize:



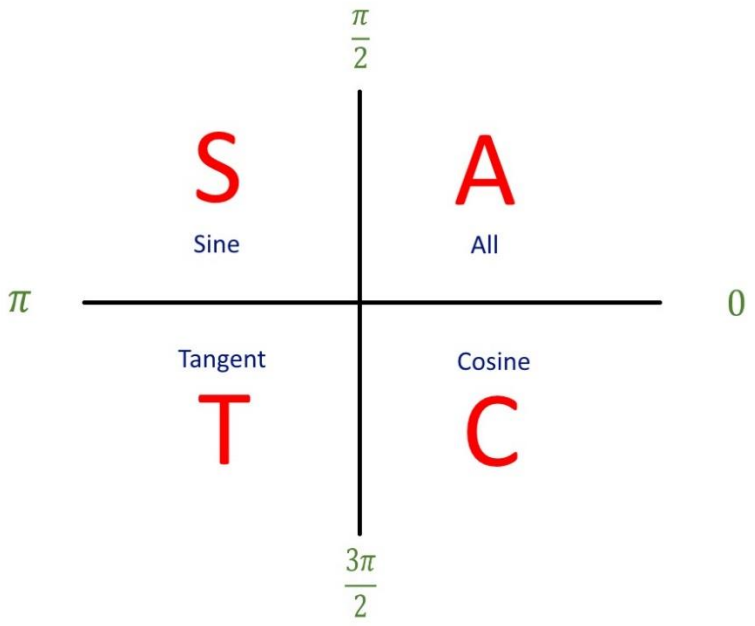
We can use SOHCAHTOA with these triangles, to evaluate trig ratios for the angles $\frac{\pi}{3}, \frac{\pi}{4}, \text{and} \frac{\pi}{6}$.

For instance, we can find that $\sin\left(\frac{\pi}{4}\right) = \frac{Opposite}{Hypotenuse} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$. $\cos\left(\frac{\pi}{6}\right) = \frac{Adjacent}{Hypotenuse} = \frac{\sqrt{3}}{2}$ and $\tan\left(\frac{\pi}{3}\right) = \frac{Opposite}{Adjacent} = \frac{\sqrt{3}}{1} = \sqrt{3}$

But, what if we have angles that are not in the first quadrant of the Cartesian plane?

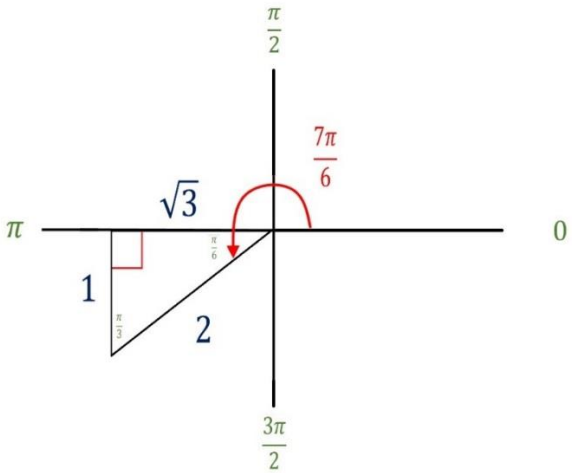
We can still use this method. It just takes a little extra work.

First, we need to remember that outside of the first quadrant, where all trig ratios are positive, the sign of the trig ratio will depend on which quadrant the angle lies in. We can use the mnemonic device CAST to help remember which trig ratios are positive and negative.



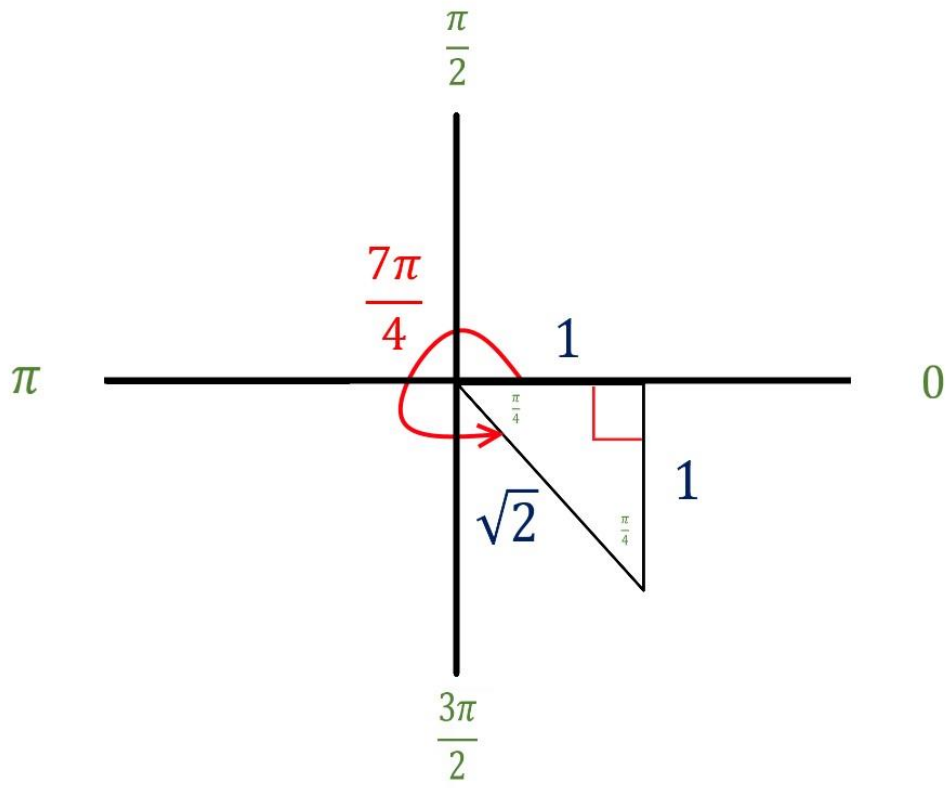
The letters of CAST correspond to which trigonometric ratio has a positive sign in that quadrant. For instance, cosine is positive in the fourth quadrant or from $\frac{3\pi}{2} \leq \theta \leq 2\pi$. Sine and tangent are negative in that quadrant. The reciprocal trigonometric ratio's sign is also determined by the quadrant, so for instance, cosecant is positive in quadrant 2 as well as sine, while cotangent and tangent are both negative there.

Let's say we want to find $\cos\left(\frac{7\pi}{6}\right)$. First, we note that we are in the third quadrant or that $\pi < \theta \leq \frac{3\pi}{2}$. Since tangent is the positive trig ratio in that quadrant, cosine must be negative. We then can use the fact that $\frac{7\pi}{6} = \pi + \frac{\pi}{6}$ and place the corresponding special triangle on the x-axis to give us the angle of $\frac{7\pi}{6}$. We can read the value of $\cos\left(\frac{7\pi}{6}\right)$ from our special triangle:



Remembering from the CAST rule that cosine is negative in the third quadrant, we use our triangle and SOHCAHTOA to determine that $\cos\left(\frac{7\pi}{6}\right) = -\frac{\sqrt{3}}{2}$. We can use this method to find $\sin\left(\frac{7\pi}{6}\right) = -\frac{1}{2}$ and $\tan\left(\frac{7\pi}{6}\right) = \frac{1}{\sqrt{3}}$ (remembering that tangent is positive in the third quadrant).

To find the trig ratios for any other special angle, it is just a matter of placing the necessary special triangle appropriately on the x-axis to obtain the required angle. For example, to find the trigonometric ratios of $\frac{7\pi}{4}$ we construct the following diagram:



Using CAST to determine that cosine is positive in the fourth quadrant, while sine and tangent are negative, we can use SOHCAHTOA and the diagram to find $\sin\left(\frac{7\pi}{4}\right) = -\frac{1}{\sqrt{2}}$, $\cos\left(\frac{7\pi}{4}\right) = \frac{1}{\sqrt{2}}$ and $\tan\left(\frac{7\pi}{4}\right) = -1$