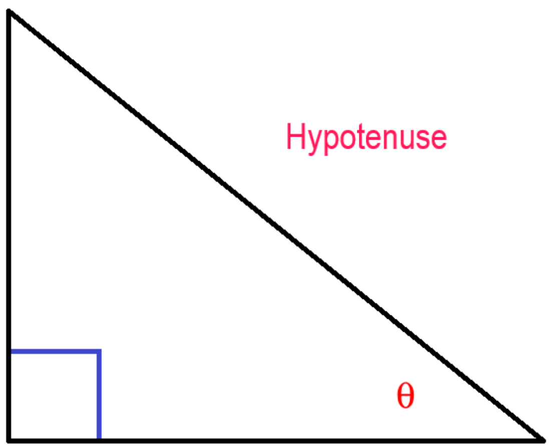
SOHCAHTOA and finding non-standard Trigonometric Ratios

Let's say we have a right-angled triangle with an angle θ. Initially we'll assume $0 \leq \theta \leq \frac{\pi}{2}$ (or that we are in the first quadrant of the Cartesian plane) We can draw the triangle and label the sides like so:



The standard trigonometric ratios are defined as being the following ratios of side lengths:

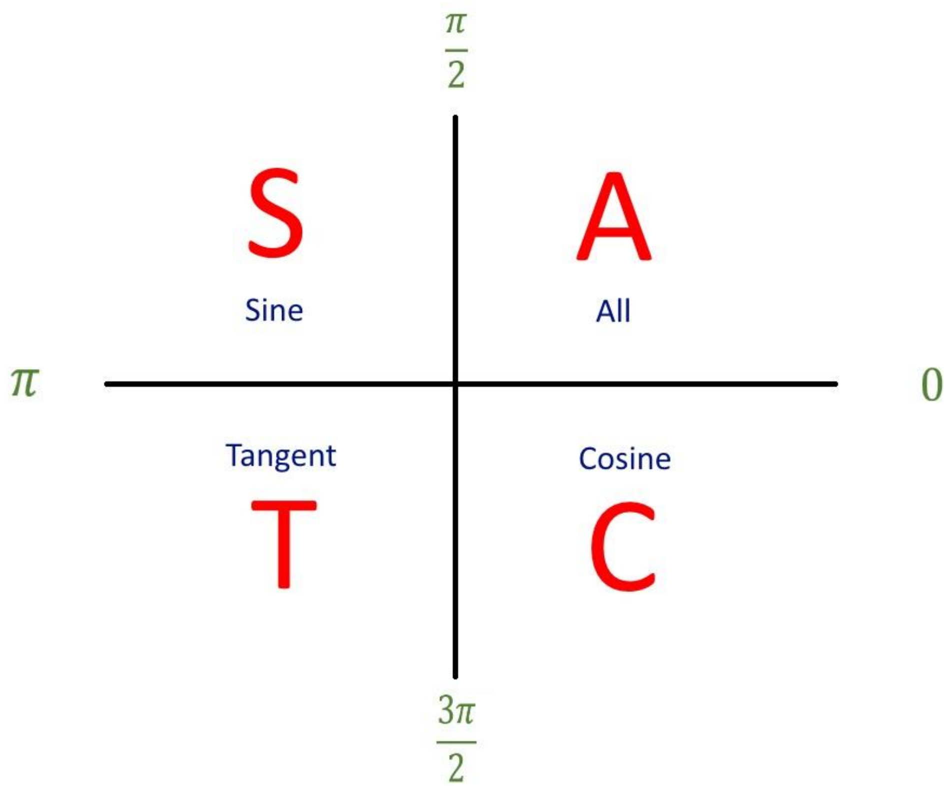
$\sin(\theta) = \frac{Opposite}{Hypotenuse}$ $\cos(\theta) = \frac{Adjacent}{Hypotenuse}$ $\tan(\theta) = \frac{Opposite}{Adjacent}$

You can use the mnemonic device SOHCAHTOA to help remember these ratios. For the "SOH" part, the "S" refers to the sine ratio, the "O" standing in for "Opposite" and the "H" for the hypotenuse. The "CAH" and "TOA" parts define cosine and tangent.

The reciprocal trigonometric ratios can be similarly defined:

$\csc(\theta) = \frac{Hypotenuse}{Opposite}$ $\sec(\theta) = \frac{Hypotenuse}{Adjacent}$ $\cot(\theta) = \frac{Adjacent}{Opposite}$

These definitions work easily while in the first quadrant of the Cartesian plane or $0 \leq \theta \leq \frac{\pi}{2}$, since all trigonometric ratios there are positive. However, if we are in any of other three quadrants, the sign of various ratios will change depending on where in the Cartesian plane they are. One way to keep track of the signs of the trigonometric ratios is to use the mnemonic device CAST.



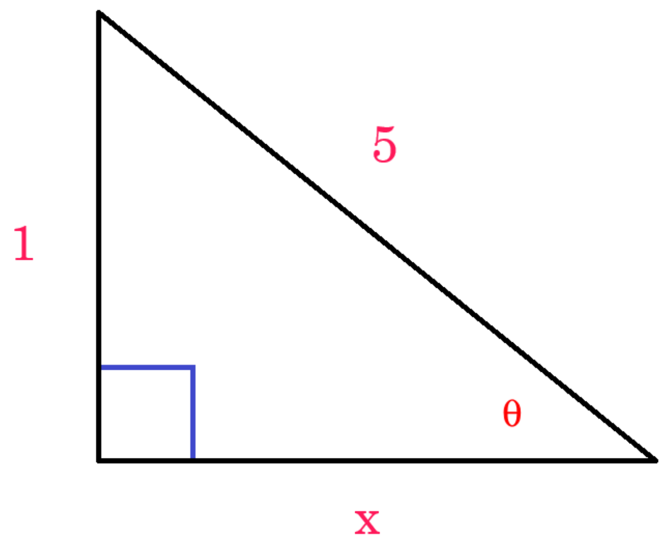
The letters of CAST correspond to which trigonometric ratio has a positive sign in that quadrant. For instance, cosine is positive in the fourth quadrant or from $\frac{3\pi}{2} \leq \theta \leq 2\pi$. Sine and tangent are negative in that quadrant. The reciprocal trigonometric ratio's sign is also determined by the quadrant, so for example, cosecant is positive in the second quadrant as well as sine, while cotangent, tangent, cosine and secant are all negative.

If we are given one trigonometric ratio, we can use the properties of right angled triangles to solve for the others.

Say that we are given that $\csc\theta = 5$ and $\frac{\pi}{2} \leq \theta \leq \pi$. We can write this as $\csc\theta = \frac{5}{1}$ which, using SOHCAHTOA would be equivalent to having a hypotenuse of length 5 and an opposite side of length 1.

Since $\sin\theta = \frac{1}{\csc\theta}$, by taking the reciprocal, we can easily find $\sin(\theta) = \frac{1}{5}$. For the other ratios, we can do the following:

First, let's look at the corresponding right-angled triangle. Using our SOHCAHTOA definition of the angles we can draw a triangle with $\csc(\theta) = 5$ (see below).



We don't know the length of the adjacent side, so it's been labelled as a variable, x. To find x, we can use the Pythagorean theorem, since the triangle that has been constructed is a right-angle triangle. . We can then sub in our known values and solve: $1^2 + x^2 = 5^2$ or $1 + x^2 = 25$ or $x^2 = 24$ which gives us $x = \sqrt{24}$.

Our next step is to use CAST to determine the signs of our other trig ratios. Since we are in the second quadrant, i.e., $\frac{\pi}{2} \leq \theta \leq \pi$, sine and cosecant are positive, but all other trig ratios are negative.

So, using our SOHCAHTOA definitions we can find $\cos(\theta) = \frac{Adjacent}{Hypotenuse} = -\frac{x}{5} = -\frac{\sqrt{24}}{5}$ and $\tan(\theta) = \frac{Opposite}{Adjacent} = -\frac{1}{x} = -\frac{1}{\sqrt{24}}$. Using our reciprocal definitions, we can also find that

$\sec(\theta) = -\frac{5}{\sqrt{24}}$ and $\cot(\theta) = -\sqrt{24}$