Modelling problems

Often we can apply mathematics to "real world" situations in the form of word problems.

Modelling problems can take on many forms, but here are a few things to try to help with their solutions:

- Know key basic formulas for area and perimeter/circumference of standard shapes. E.g. rectangles, squares, circles and triangles.
- Read the question carefully. When a value is given, try and relate it to a particular variable. Look for radius, areas, volumes, lengths of sides, height, number of items purchased, etc.
- Draw a picture to help (if it's applicable). Make sure to label it with the values given in the word problem.
- Try and find a formula which answers (or helps to answer) the question and matches the variables you are given. Sometimes you may need more than one formula. For instance, the Example 1 below uses both perimeter and area of a square.
- Read the question carefully again to make sure your model and calculations are answering what the question asked.
- Finally, include proper units (if appropriate) in your answer.

Example 1

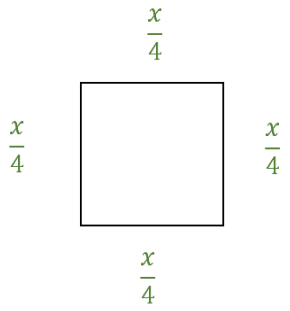
Say you wanted to build a square fenced-in yard for your pet. The fencing material is sold by the meter.

a. *Write a function that determines the area of your dog pen in terms of the number of meters of fencing purchased*

We are asked to find area as a function of the number of meters.

Step one Let x be the number of meters purchased. This length, x, forms the perimeter of the square. Since a square has four sides of equal length, the perimeter of a square is $x = 4s$where s is the length of the side of our square.

We can rewrite this as $s = \frac{x}{4}$ and label our diagram below.



The area of a square is the length of its side squared or

$$A = s^2 = \left(\frac{x}{4}\right)^2 = \frac{x^2}{16}$$

Which gives us a formula for the area in terms of x meters of fencing purchased.

b. If you purchase 10 meters of fencing, what is the area of your dog pen?

We are given $x = 10\ m$ and asked to find A. So, we evaluate our function at x= 10

$A(10) = \frac{10^2}{16} = \frac{100}{16} = \frac{25}{4}\ m^2$ is the area of the dog pen.

Make sure that you have the correct units! Area (in this case) is in square meters.

Example 2

A triangle has a height that is 2 more than 3 times its base and a total area of 60 cm². Find the base and height.

We are given the area of a triangle is 60 square units or $A = 60$. We are also given that the triangle's height (h) is 2 more than 3 times its base (b) or $h = 2 + 3b$

Since we know that the area of a triangle is $A = \frac{1}{2}bh = 60$, and that $h = 2 + 3b$ we can write

$$A = 60 = \frac{1}{2}b(2 + 3b)$$

We can rewrite this as $60 = \frac{1}{2}(2b + 3b^2) = b + \frac{3}{2}b^2$

Or $\frac{3}{2}b^2 + b - 60 = 0$

$$\frac{1}{2}(3b^2 + 2b - 120) = 0$$

Which we can factor to get $\frac{1}{2}(3b + 20)(b - 6) = 0$

This implies that either $b = 6$ *or* $b = -\frac{20}{3}$. Since we cannot have a triangle with a negative length for a base, only $b = 6$ is a valid answer. Remember that you need to check if your solution(s) are valid for the original equation or application you are given!

To find h, we sub our b value back into $h = 2 + 3b = 2 + 3(6) = 2 + 18 = 20$.

The triangle we are looking for has a base of 6 cm and height of 20 cm.

Example 3

For a grocery supplier, the initial per unit price of a 5kg box of nutmeg is $80.00 but the price decreases by 25 cents for every box sold, up to 200 boxes. Find the total amount of revenue that can be made by selling x boxes, where $0 \leq x \leq 200$. Then find the revenue made by selling 75 boxes of nutmeg.

Case 1: Selling one box. At $x = 1$, the price per box is $\$80.00$ and we sell 1 box
The revenue is the price per box ($80) multiplied by the number of boxes sold (1)

So, if we call our function $R(x)$ for revenue, $R(1) = 80(1) = \$80$.

Case 2: Selling two boxes or $x = 2$. The price per box for 2 boxes is $\$80 - 0.25(1) = \$79.75$ and we sell 2 boxes. Thus $R(2) = 79.75(2) = \$159.50$.

Case 3: $x = 3$. The price per box is $\$80 - 0.25(2) = \$79.50$ and we sell 3 boxes. Thus $R(3) = 79.50(3) = \$238.50$

For selling x boxes, we know that $R(x) =$ (the price per box) × (number of boxes sold),
The number of boxes sold is x, and if we look at the pattern, the price per box is
$80 - 0.25$ (number of boxes sold – 1) $= 80 - 0.25(x - 1)$

So, we have $R(x) = (80 - 0.25(x - 1))x$ which simplifies to $R(x) = 80.25x - 0.25x^2$

To answer the second question, we need to find $R(x)$ when $x = 75$. We simply calculate $R(75) = \$4612.60$