

# **The application of copula function modelling to Bordeaux en primeur wine ratings**

**Don Cyr**

**Professor of Finance**

**Goodman School of Business, Brock University**

**Fellow, Cool Climate Oenology and Viticulture Institute**

**Lester Kwong**

**Associate Professor of Economics**

**Brock University**

**Fellow, Cool Climate Oenology and Viticulture Institute**

**Ling Sun**

**Assistant Professor of Economics**

**Brock University**



Brock University

# Introduction

- Bordeaux en primeux process
- Impact of wine critic ratings on wine prices
- Copula functions and their use in modelling nonlinear dependence
- Data
- Results – Robert Parker and en primeur prices
- Results of Parker/Martin analysis
- Future applications for Copula Function modelling

# En Primeur Process

## The Bordeaux En Primeur Process

- Existed in France for centuries as a form of futures market
- Spring of each year, after the prior harvest, merchants, wine critics and trade associations gather to taste and rank barrel samples of wines that are frequently eight to ten months old
- Wine is then sold ahead of bottling and ultimate release of the vintage, which may be up to two years later
- Benefit to Purchaser - provides the opportunity for the purchaser to secure a vintage before it is bottled and released, typically at a much lower price
- Benefit to Producer - cash flow prior to the release and sale of the wine in the retail market
- Uncertainty - the chateau must decide how much wine to allocate to futures sales as opposed to the retail market, when the wine is bottled and released
- Risk is mitigated the higher the en primeur price, and prices have been shown to be heavily dependent on the critic barrel scores achieved

# Wine Critic Barrel Ratings

## Impact of Parker Barrel Ratings

En primeur prices appear to be heavily dependent upon the ranking of the wine based on the barrel tastings, despite the uncertainty remaining, associated with the continued aging process

It has long been known in the Bordeaux en primeur market that the barrel scores of the prestigious wine critic Robert Parker Jr. have had a great influence on the en primeur price offerings by the chateaux

Parker's ratings have been largely viewed as the authority on Bordeaux en primeur wines

**Noparumpa et al. (2015), Ali et al. (2010), Ashenfelter, (2010), Jones and Storchmann, (2001).**

# Wine Critic Barrel Ratings

## Impact of Wine Critics Ratings on Wine Prices

- A fairly large body of literature deals with the impact of the ratings of wine critics on the demand for wine and wine prices. Studies of this nature have been carried out for wines originating from several countries and over different time periods
- “Over 60 studies and 180 hedonic wine price models over a 20 year period.....”
- “The research identifies that the relation between the price of wine and its sensory quality rating is a moderate partial correlation of +0.30.”

**Oczkowski, E., & Doucouliagos, H. (2015). Wine prices and quality ratings: A meta-regression analysis. *American Journal of Agricultural Economics*, 97(1), 103-121.**

# Wine Critic Barrel Ratings

Noparumpa, T., Kazaz, B., and Webster, S. (2015), “Wine futures and advanced selling under quality uncertainty”, *Manufacturing & Service Operations Management*. 17(3), 1-16

Notes some non-linearity in the relationship of Parker ratings and wine prices

**Model Risk**– Risk due to assumptions regarding the fundamental dependence structure between variables and its stationarity.

Generally a regression analysis is used, assuming the dependence structure is captured fairly well by linear correlation.

It appears that this is not often the case.

One solution to the issue is the use of copula functions to fit multivariate distributions, incorporating nonlinear dependence

Useful for capturing “tail dependence” – higher correlation at the “tails” of the univariate (marginal) distributions comprising the multivariate distribution

# COPULA Functions

## Based upon Sklar's Theorem (1959)

If  $F$  is a joint distribution function of  $m$  random variables  $(y_1, \dots, y_m)$  with marginal distributions  $F_1, \dots, F_m$

Then there exists an  $m$ -dimensional copula  $C: [0,1]^m \rightarrow [0,1]$  (from the unit  $m$ -cube to the unit interval) which satisfies the following conditions:

1.  $C(1, \dots, 1, a_n, 1, \dots, 1) = a_n$  for every  $n \leq m$  and for all  $a_n$  in  $[0,1]$

If the realizations of  $m-1$  variables are known, each with a probability of one, then the joint probability of the  $m$  outcomes is the same as the probability of the remaining uncertain outcomes.

2.  $C(a_1, \dots, a_m) = 0$  if  $a_n = 0$  for any  $n \leq m$

The joint probability of all outcomes is zero if the marginal probability of any outcome is zero.

3.  $C$  is  $m$ -increasing

$C$ -volume of any  $m$ -dimensional interval is non-negative.

# COPULA Functions

## Sklar's Theorem (1959)

Given  $F(y_1, \dots, y_m)$  with univariate marginal distributions  $F_1(y_1), \dots, F_m(y_m)$  and inverse functions  $F_1^{-1}, \dots, F_m^{-1}$ , then

$$y_1 = F_1^{-1}(u_1) \sim F_1, \dots, y_m = F_m^{-1}(u_m) \sim F_m$$

Where  $u_1, \dots, u_m$  are uniformly distributed variates.

$$\begin{aligned} F(y_1, \dots, y_m) &= F(F_1^{-1}(u_1), \dots, F_m^{-1}(u_m)) \\ &= \Pr[U_1 \leq u_1, \dots, U_m \leq u_m] \\ &= C(u_1, \dots, u_m) \end{aligned}$$

Is the unique copula function associated with the distribution function and

$$(F_1(y_1), \dots, F_m(y_m)) \sim C$$

and if  $U \sim C$ , then

$$(F_1^{-1}(u_1), \dots, F_m^{-1}(u_m)) \sim F$$

***Essentially Copulas can be used to express a multivariate distribution in terms of its marginal distributions!***



# COPULA Functions

## Sklar's Theorem (1959)

For an  $m$ -variate function  $F$ , the copula associated with  $F$  is a distribution function  $C:[0,1]^m \rightarrow [0,1]$  that satisfies.

$$F(y_1, \dots, y_m) = C(F_1(y_1), \dots, F_m(y_m); \theta)$$

Where  $\theta$  is a vector of parameters called the dependence parameter which measures dependence between the marginal distributions.

In bivariate applications  $\theta$  is typically a scalar.

***The joint distribution is expressed in terms of its respective marginal distributions and a function  $C$  that binds them together. This allows for the consideration of marginal distributions and dependence as two separate but related issues.***

# COPULA Functions

## Application of Copula Functions

For a variety of reasons, largely due to the high dimensionality of  $m \geq 3$  copula estimation, most research has focused on bivariate parametric copulas.

### Parametric copulas

- Implicit (Gaussian and Student t copula) – implied by known multivariate distribution functions and do not have simple closed forms.
- Explicit (Archimedean Copulas) – simple closed forms.

### Form and relationship of parameters to Spearman correlation

Copula	$C(u_1, u_2)$	$\theta$ - domain	Spearman's $\rho$
Gaussian	$\Phi_G[\Phi^{-1}(u_1), \Phi^{-1}(u_2); \theta]$ $\Phi$ is the standard normal distribution $\Phi_G(u_1, u_2)$ is the standard bivariate normal distribution	$-1 \leq \theta \leq +1$	$\frac{6}{\pi} \arcsin\left(\frac{\theta}{2}\right)$
Clayton	$(u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta}$	$\theta \in (0, \infty)$	complicated
Frank	$-\theta^{-1} \log \left[ 1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{(e^{-\theta} - 1)} \right]$	$\theta \in (-\infty, \infty)$	$1 - \frac{12}{\theta} [D_1(\theta) - D_{12}(\theta)]$ $D_k(x)$ is the Debye function
<u>Gumbel</u>	$\exp(-(-\log u_1^\theta - \log u_2^\theta)^{1/\theta})$	$\theta \in (1, \infty)$	complicated

Trivedi, P.K. and Zimmer, D.M. (2005) Copula Modeling: An Introduction for Practitioners

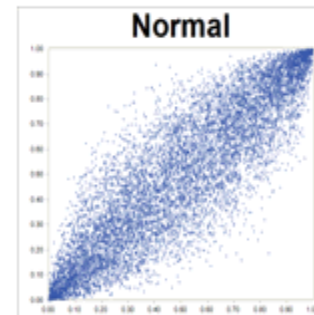
# COPULA Functions

**Two Parametric Families of Copula Functions are commonly used.**

## **1. ELLIPTICAL COPULAS**

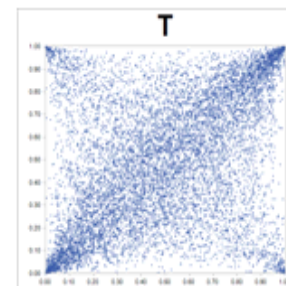
Can capture some degree of tail dependence but are limited in that they are symmetric. Tend to under estimate tail dependence if it is asymmetric.

### **Gaussian (Normal) Copula**



### **Student-T Copula**

More flexible than the Gaussian copula because It does not assume that uncorrelated variables are independent.

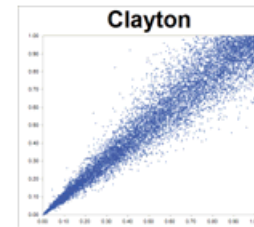


# COPULA Functions

**ARCHIMEDEAN COPULAS**– allow for a wider variety of dependence structures, particularly asymmetric

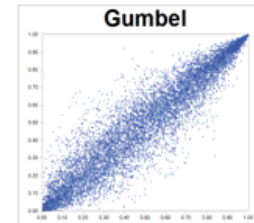
## **Clayton Copula**

Greater dependence in the lower tail.



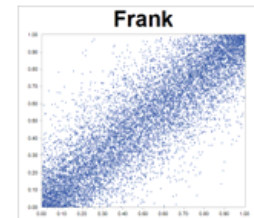
## **Gumbel Copula**

Greater dependence in the upper tail.



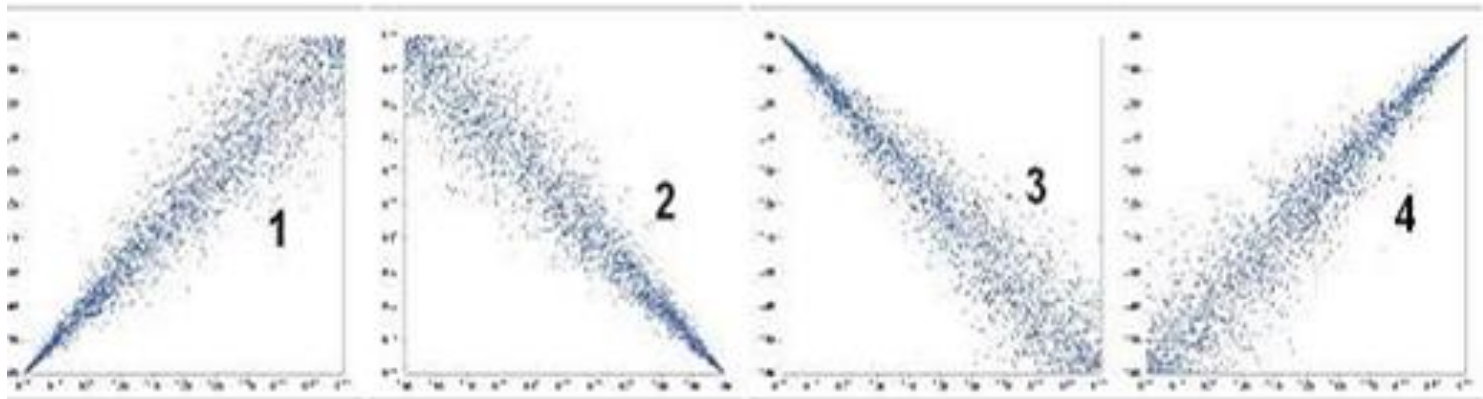
## **Frank Copula**

Greater correlation in the middle section than in the tails.



## COPULA Functions

Clayton and Gumbel Copulas can also be estimated as transformations of the variables  $(u, v)$  by taking one or both of the variables and transforming them as  $1-u$  and/or  $1-v$ , resulting in three additional patterns that can be tested. This provides for directional patterns of 1, 2, 3 and 4.



## COPULA Functions – A side History

Mathematics of Copula Functions developed in 1959 by Sklar

### **First application in Financial Economics:**

Embrechts, P., A. McNeil, and D. Straumann (1999). Correlation and dependence in risk management: Properties and pitfalls. *RISK*, May 1999, 69–71

### **2008 Financial Crisis**

Seminal article that led to the development of Collateralized Debt (Mortgage) Obligations (CDO's):

Li, D. X. (2000). On Default Correlation: A Copula Function Approach. *The Journal of Fixed Income*, 9(4), 43-54.

Interesting connection between copula function modelling and the 2008 Financial Crisis - the incorrect use of the Gaussian copula to model CDO's comprised of multiple mortgages:

Salmon, F. (2009). Recipe for Disaster: The Formula That Killed Wall Street, *Wired Magazine*

## Ratings and En Primeur Price Data

Database of en primeur prices along with wine critics ratings 2004 – current

<http://www.bordoverview.com>

Bolomey Wijnimport Amsterdam – wine sellers

2004 through 2010 was chosen as the period of study as it reflects a time period starting from the renown 2005 harvest and carrying through 2010 of a stable sustained bull run in futures prices. It has been alluded to that Parker's barrel ratings had a significant impact on rising en primeur prices. After 2010 (until 2014) lower sales plagued the market along with downward pressure on prices.

In addition 2003 Parker's barrel ratings were released after the en primeur prices were set by chateaux (Ali et al., 2010)

# Data and Analysis

**Data is also provided for LEFT Bank (south of the Gironde and Garonne rivers - Cabernet Sauvignon dominant) and RIGHT bank (north of the Gironde and Dordogne rivers - Merlot dominant) wines**

Jones and Storchman (2001) - Show less sensitivity of wine prices to Parker ratings in the case of wines with a higher share of Merlot grape than Cabernet Sauvignon.

Support Bordoverview: buy your [Bordeaux 2015](#) primeurs at Bolomey Wijnimport Amsterdam.  
Read about Bordeaux 2015 on [Bolomey Blog](#). Follow [Bordoverview on Twitter](#).

← Overview settings: 2010 ▾ Left bank ▾ Update

Wine	Year	AOC	Class. ▲	Size	By	RP	NM	JR	TA	BD	JS	JL	De	RVF	JA	PW	RG	Price	+/-
Haut-Brion	2010	Pessac-Léognan	1st GCC	43	Mascléf, J-P	98-100	96-98	18++	·	·	97-98	·	19.5	18.75	98-99	5	19	€ 925	+9%
Lafite-Rothschild	2010	Pauillac	1st GCC	103	Boissenot, J.	98-100	95-97	19	·	·	100	·	20	20	99	5	20	€ 1300	+44%
Latour	2010	Pauillac	1st GCC	67	Boissenot, J.	98-100	96-98+	19	·	·	98-99	·	20	·	98	5	19	€ 1150	+28%
Margaux	2010	Margaux	1st GCC	81	Boissenot, J.	96-98	97-99	19	·	·	100	·	20	20	99-100	5	20	€ 950	+12%
Mouton-Rothschild	2010	Pauillac	1st GCC	83	Boissenot, J.	97-100	98-100	18.5	·	·	99-100	·	19.5	19.75	99-100	4½	19	€ 925	+9%
Brane-Cantenac	2010	Margaux	2nd GCC	90	Lurton, H.	93-96	94-96	17	·	·	91-92	·	18	17	92	4	17	€ 76	+25%
Cos d'Estournel	2010	St-Estèphe	2nd GCC	64	Boissenot, J.	95-97	96-98	18.5	·	·	96-97	·	19	19.25	94-95	4½	19	€ 273	-6%
Ducru-Beaucaillou	2010	St-Julien	2nd GCC	55	Boissenot, J. & E.	96-98+	95-97	18	·	·	99-100	·	19	19.5	98	5	19	€ 207	-17%
Durfort-Vivens	2010	Margaux	2nd GCC	30	Lurton, G.	89-91	89-91	16	·	·	91-92	·	18	15.5	90	3½	17	€ 45	+29%
Gruaud-Larose	2010	St-Julien	2nd GCC	82	Pauli, G./Boissenot	92-94	92-94+	16	·	·	93-94	·	18	18.5	94+	4½	17	€ 63	+13%
Lascombes	2010	Margaux	2nd GCC	84	Rolland, M.	94-97	·	17.5	·	·	91-92	·	17.5	17	93+	4½	19	€ 100	+19%
Léoville-Barton	2010	St-Julien	2nd GCC	47	Boissenot, J.	91-93+	96-98	17.5+	·	·	97-98	·	18.5	18.75	94-95	4½	18	€ 100	+15%

**Copula function models were estimated for each of the years 2004 – 2010 and for left and right bank in each case, using Vose ModelRisk software.**  
**[www.vosesoftware.com](http://www.vosesoftware.com)**



# Goodness of Fit Tests for Copulas

## **Standard Approach to Copula Function Modelling:**

Fit several copula functions to the data and apply maximum likelihood goodness-of-fit tests to see which function models the dependency structure relatively better.

Information Criteria Tests (varying penalties for additional parameters)

Akaike Information Criteria (AIC)

Bayesian (Schwartz) Information Criteria (BIC)

Hannan-Quinn Information Criteria (HQIC)

Problem is that they do not provide the power of the decision rule.

# Goodness of Fit Tests for Copulas

A few goodness of fit tests have recently been developed for copula functions but significant issues still remain:

*Problematic due to the high dimensionality of the problem.*

- Full multivariate approach - Panchenco (2005) – *Physica A*

*Consequently there are approaches that attempt to reduce the problem from a multivariate to a univariate problem:*

- Berg and Batten (2005) – *Norwegian Computing Centre*
- Genest, Quessy and Remillard (2006) – *Scandinavian Journal of Statistics*

*However the power of the tests appear to differ with sample size, dimensionality and copula function being tested:*

- Berg, D. (2009). Copula goodness-of-fit testing: an overview and power comparison. *The European Journal of Finance*, 15(7-8), 675-701
- Fermanian, J. D. (2013). An overview of the goodness-of-fit test problem for copulas. In *Copulae in Mathematical and Quantitative Finance* (pp. 61-89). Springer Berlin Heidelberg.

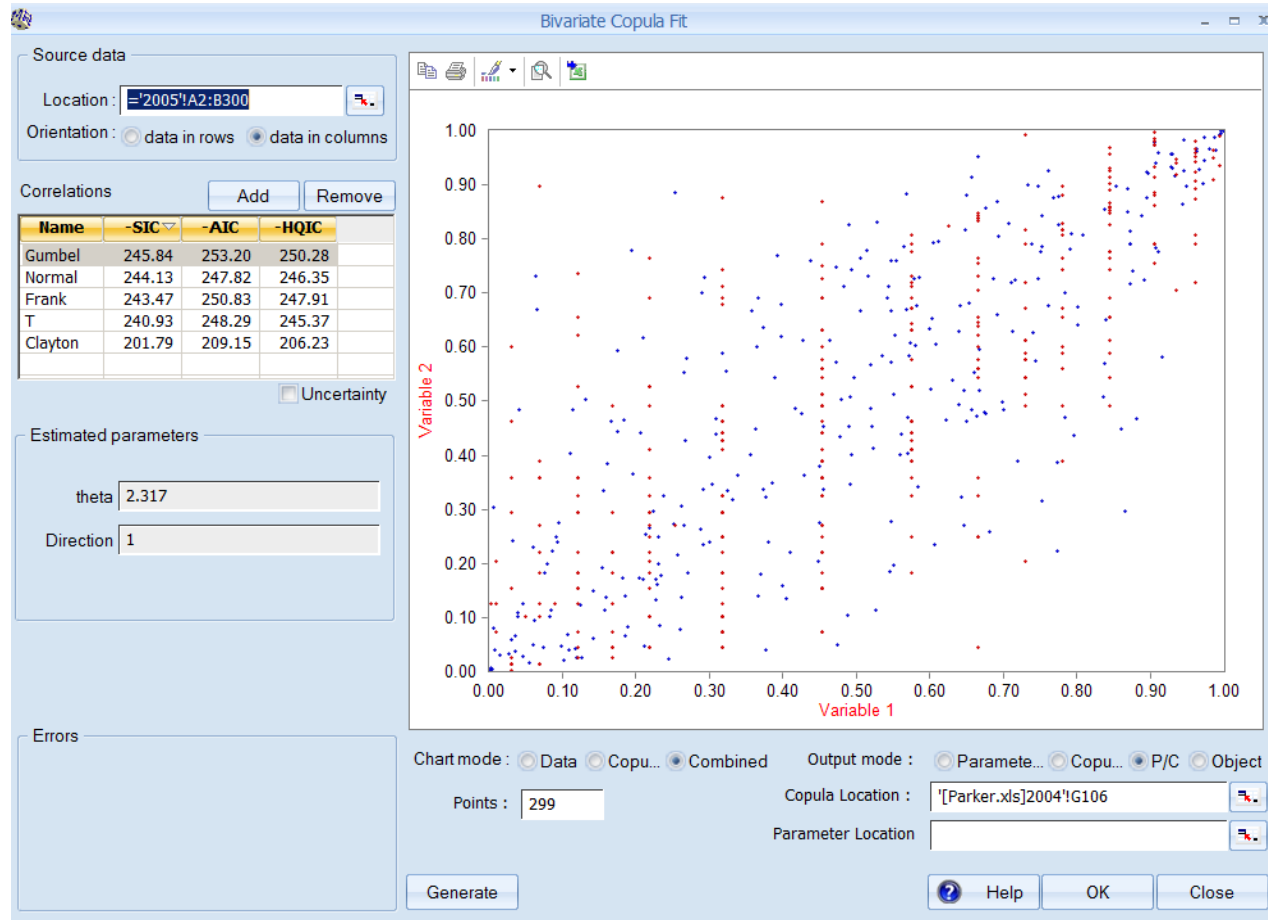
# Results

Used *ModelRisk* Software to estimate traditional Copula Functions – maximum likelihood estimation of copula function parameters used to identify best fit.

- Clayton
- Frank
- Gumbel
- Normal
- T

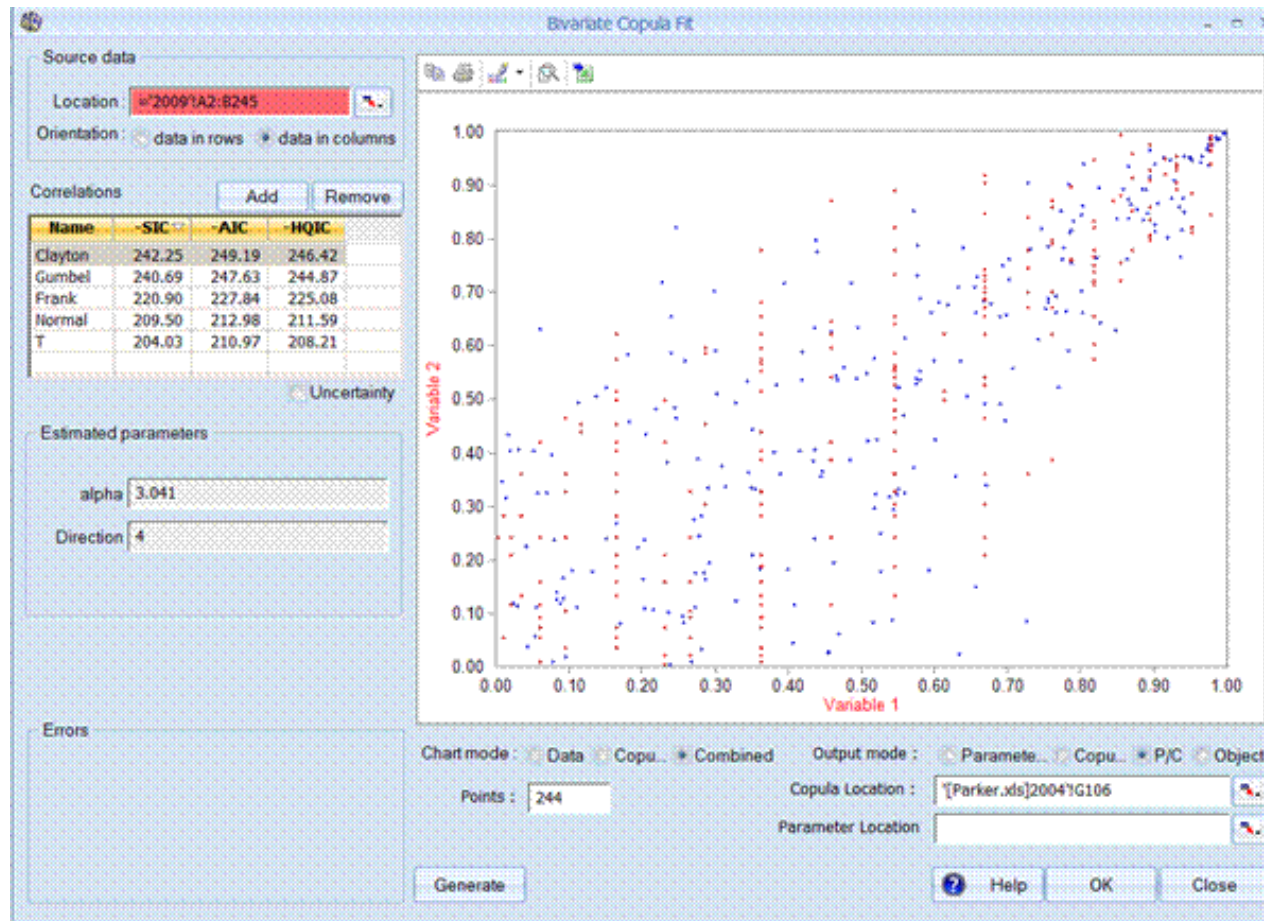
# Results

2005 example (combined left and right bank) – Gumbel copula provided best fit identifying tail dependence in high values



# Results

2009 example (combined left and right bank) – Clayton copula (direction 4) provided best fit. Again capturing tail dependence at higher values.



# Results

*Table 2*  
**Maximum Likelihood Test Results for Best Fitting Copula**

	Left Bank							Right Bank						
	Copula	obs	$\tau$	$\rho_P$	$\theta$	$\lambda_U$	$\lambda_L$	Copula	obs	$\tau$	$\rho_P$	$\theta$	$\lambda_U$	$\lambda_L$
2004	Gumbel	80	0.52	0.63	2.14	0.62	0.00	Gumbel	81	0.47	0.48	1.88	0.55	0.00
2005	Gumbel	150	0.57	0.58	2.33	0.65	0.00	Frank	149	0.55	0.35	6.77	0.00	0.00
2006	Clayton (4)	98	0.52	0.51	2.16	0.00	0.73	Clayton (4)	85	0.51	0.59	2.07	0.00	0.71
2007	Clayton (4)	91	0.45	0.42	1.63	0.00	0.65	Clayton (4)	98	0.37	0.37	1.19	0.00	0.56
2008	Gumbel	105	0.63	0.68	2.67	0.70	0.00	Clayton (4)	109	0.58	0.57	2.75	0.00	0.78
2009	Clayton (4)	114	0.63	0.63	3.38	0.00	0.81	Gumbel	130	0.59	0.51	2.43	0.67	0.00
2010	Gumbel	110	0.61	0.63	2.57	0.69	0.00	Gumbel	115	0.63	0.55	2.71	0.71	0.00
2004-10	Gumbel	748	0.55	0.52	2.24	0.64	0.00	Gumbel	767	0.51	0.42	2.03	0.59	0.00

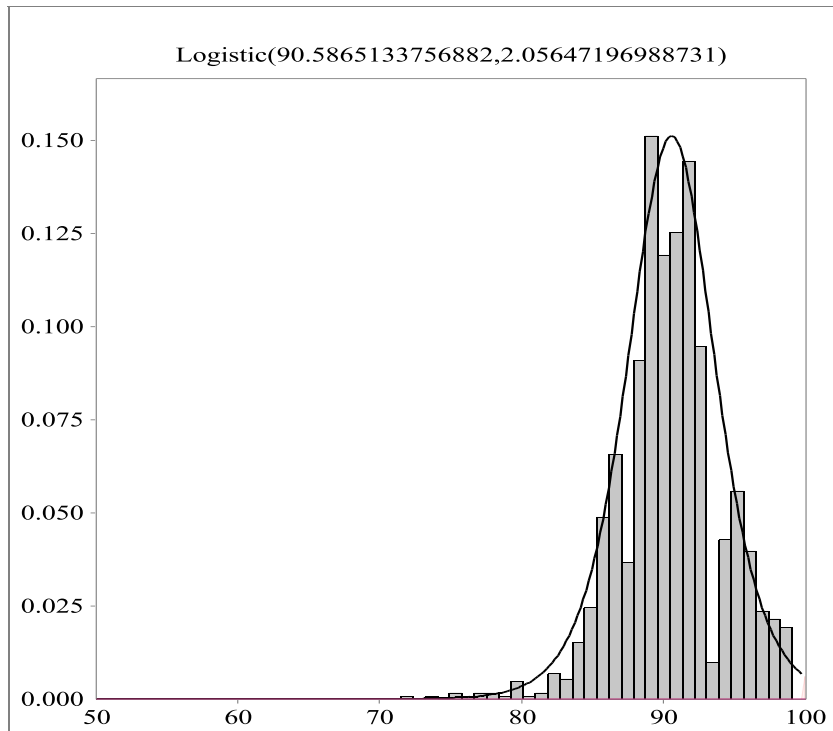
		Combined Left and Right Bank					
	<b>Copula</b>	<b>obs</b>	$\tau$	$\rho_P$	$\theta$	$\lambda_U$	$\lambda_L$
<b>2004</b>	Gumbel	161	0.49	0.49	1.97	0.58	0.00
<b>2005</b>	Gumbel	299	0.57	0.39	2.32	0.65	0.00
<b>2006</b>	Clayton (4)	183	0.51	0.54	2.05	0.00	0.71
<b>2007</b>	Clayton (4)	189	0.40	0.39	1.35	0.00	0.60
<b>2008</b>	Gumbel	214	0.60	0.53	2.49	0.68	0.00
<b>2009</b>	Clayton (4)	244	0.60	0.58	3.04	0.00	0.80
<b>2010</b>	Gumbel	225	0.61	0.57	2.58	0.69	0.00
<b>2004-10</b>	Gumbel	1515	0.53	0.45	2.14	0.62	0.00

# Results

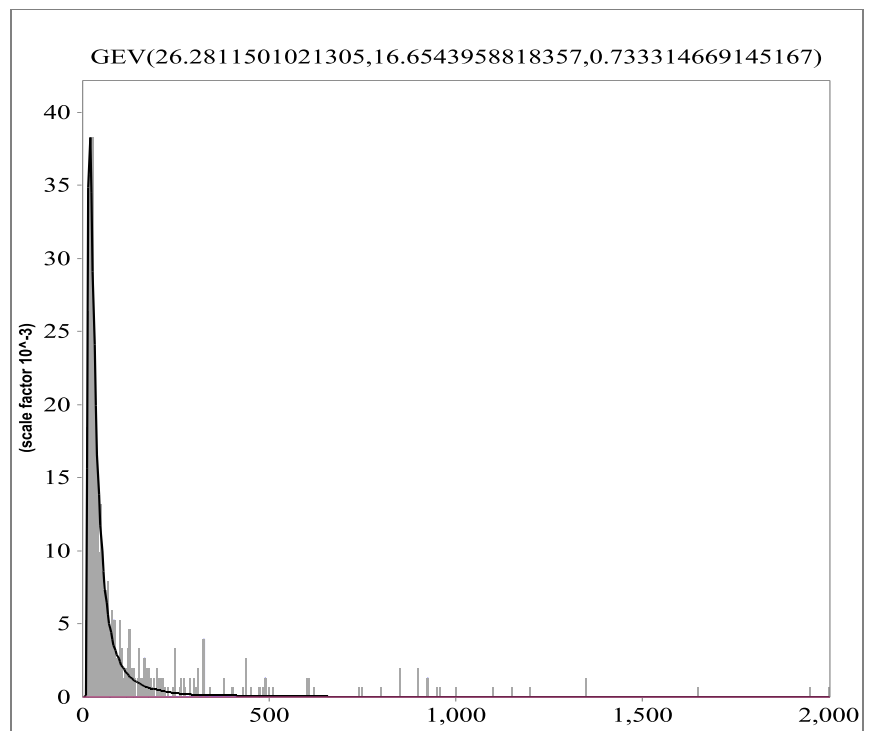
Once the appropriate copula function is identified, the marginal distributions can be separately identified.

**Figure 4: Histogram and Plot of Best Fitting Distribution for Parker Ratings and Wine Prices: 2004-10 Left and Right Bank**

*Parker Ratings and Logistic Distribution*



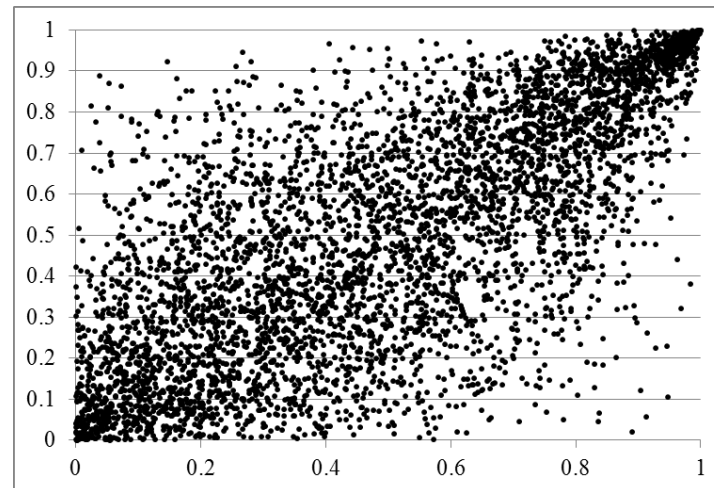
*Wine Prices and GEV Distribution*



# Results

Given the copula function and the marginal distributions we can then use Monte Carlo simulation to generate ratings and prices from a bivariate distribution that allows us to generate probabilities. We used Monte Carlo simulation to generate 5,000 combinations of ratings and prices

**Figure 5: Bivariate Uniform Distribution Plot of Simulated Parker Ratings and Price Data**

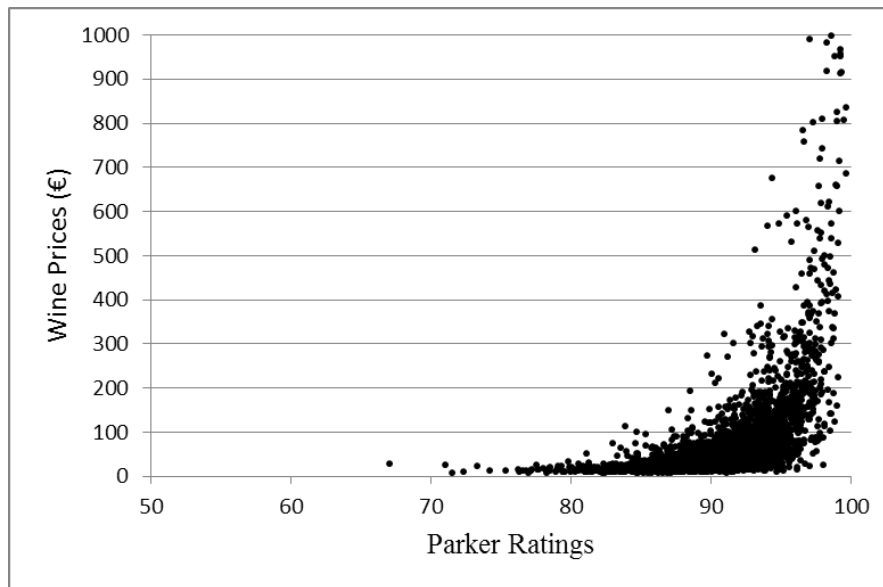




# Results

Given the copula function and the marginal distributions we can then use Monte Carlo simulation to generate ratings and prices from a bivariate distribution that allows us to generate probabilities. We used Monte Carlo simulation to generate 5,000 combinations of ratings and prices

**Figure 6: Graph of Simulated Parker Ratings and Wine Prices**



Rating	Average Price	Standard Deviation	Pearson Correlation
75-80	15.83 €	5.81 €	0.27
80-85	19.42 €	12.05 €	0.15
85-90	27.55 €	16.93 €	0.21
90-95	59.27 €	63.68 €	0.36
95-100	391.96 €	779.63 €	0.52

## Future Research

Database also consists of rankings by other wine critics

Neal Martin

Jancis Robinson

Tim Atkin

Michel Bettane and Thierry Desseauve

James Suckling

Jeff Leve

Decanter wine magazine critics

La Revue du Vin de France

Jane Anson

Perswin

Rene Gabriel

## Future Research

Ashton, R. H. (2012). Reliability and Consensus of Experienced Wine Judges: Expertise Within and Between? *Journal of Wine Economics*, 7(01), 70-87.. - Mean reliability between judges is .5 across various studies.

Cardebat, J. M., & Livat, F. (2016). Wine experts' rating: a matter of taste?. *International Journal of Wine Business Research*, 28(1), 43-58. – Variation might be explained by taste preferences of critics

Multivariate copula function could be attempted using addition expert rating or combining ratings:

Cardebat, J. M., & Paroissien, E. (2015). Standardizing expert wine scores: An application for Bordeaux en primeur. *Journal of Wine Economics*, 10(03), 329-348.

## In Progress

**February 2015** After 38 years, Parker announced that he would no longer review Bordeaux wine futures; turning the responsibility over to his successor Neal Martin, a British wine critic.

Martin – a wine blogger who started the website *Wine Journal* in 2003 gained a substantial following over a short period of time and joined Parker's prestigious publication, the *Wine Advocate* as a wine writer and critic in 2006.

**April 2016** - Martin assumed responsibility for the review of all Bordeaux wines, both in barrel and bottle, for the *Wine Advocate*

## In Progress

### Issue

Parker is credited with having had a significant impact on Bordeaux wines  
Pushed the industry to invest in new technology and equipment resulting in greater consistency over the years

Not without some controversy – Parker has been criticised for advocating style over substance, resulting in a homogenous world of highly oaked and over-extracted wines.

Appointment of Martin creates some uncertainty for many chateaux, both with respect to the future influence of Martin's ratings and their consistency, or lack thereof, with that of Parker's.

## Parker and Martin

*en primeur* wine database [www.borderview.com](http://www.borderview.com)

For the period of 2010 through 2012, Robert Parker and Neal Martin independently rated many of the same Bordeaux *en primeur* wines, providing the opportunity to examine the bivariate distributional relationship between their evaluations.

Provides for 325 left bank concurrent wine ratings and 332 in the case of the right bank, over the three year period.

it has been noted that both critics have expressed a preference for Merlot dominated blends stemming from Bordeaux right bank wines

Both critics use the same Parker rating system of 50 – 100.

## Parker and Martin

### Best Fitting Copula Functions\* Employing Akaike Information Criteria Test Statistic

	Left Bank						Right Bank					
	Copula	obs	$T_K$	$\rho_P$	$\lambda_U$	$\lambda_L$	Copula	obs	$T_K$	$\rho_P$	$\lambda_U$	$\lambda_L$
<b>2010</b>	Normal	114	0.49	0.69	0.00	0.00	Clayton(-)	107	0.47	0.68	0.68	0.00
<b>2011</b>	Clayton(-)	98	0.39	0.61	0.58	0.00	Normal	117	0.33	0.68	0.00	0.00
<b>2012</b>	Clayton(-)	113	0.38	0.52	0.57	0.00	Normal	108	0.45	0.67	0.00	0.00
<b>2010-12</b>	Clayton(-)	325	0.47	0.67	0.67	0.00	Gumbel	332	0.43	0.64	0.52	0.00

	Combined Left and Right Bank					
	Copula	obs	$T_K$	$\rho_P$	$\lambda_U$	$\lambda_L$
<b>2010</b>	Gumbel	221	0.47	0.68	0.55	0.00
<b>2011</b>	Clayton (-)	215	0.35	0.53	0.53	0.00
<b>2012</b>	Gumbel	221	0.44	0.62	0.53	0.00
<b>2010-12</b>	Gumbel	657	0.45	0.65	0.53	0.00

## Parker and Martin

Significant tail dependence in the multivariate distribution of Parker's and Martin's ratings, particularly for left bank wines.

2011, 2012: Martin's ratings of left bank wines appear to be highly correlated with that of Parker's when the ranking is high (upper tail dependence), but less so at the lower range.

The right bank exhibits a different correlation pattern.

2010 – upper tail dependence

2011, 2012. - Gaussian (Normal) copula - lack of tail dependence

Did Martin start to develop his own idiosyncratic preferences in terms of Bordeaux wines and particularly highly ranked right bank wines?

If so, does this add risk for Bordeaux wine producers?



## Other Areas of Research with Copula Functions

**Increased use of Copula functions in Agricultural Economics for the modelling of the relationship between weather variables, prices and crop yields**

Vedenov (2008) ) - *Application of copulas to estimation of joint crop yield distributions*

Woodward et al. (2011) - *Impact of copula choice on the modeling of crop yield basis risk*

Bokusheva (2011) - *Measuring dependence in joint distributions of yield and weather variables*

Okhrin et al., (2013) - *Systemic weather risk and crop insurance: the case of China*

Boziac et al. (2014) - *Tails Curtailed: accounting for nonlinear dependence in pricing margin insurance for dairy farmers*

Bokusheva et al (2016). *Satellite-based vegetation health indices as a criteria for insuring against drought-related yield losses*

Cyr, D., Eyler, R., & Visser, M. (2013). *The Use of Copula Functions in Pricing Weather Contracts for the California Wine Industry*. Working paper. Brock University

# The End

