

# BROCK UNIVERSITY MATHEMATICS MODULES

## 12D1.1: Introduction to Rates of Change

Author: Angela Coleman

### WWW

- What it is: The rate at which function values change with respect to changes in the independent variable. If the independent variable is time, then the rate of change tells us how fast function values are changing.
- Why you need it: Rate of change is often the most important quantity for describing quantities that are changing.
- When to use it: When you want to describe how fast changes are occurring, especially in modelling real-world applications.

### PREREQUISITES

Before you tackle this module, make sure you have completed these modules:

**Slope**

### WARMUP

Before you tackle this module, make sure you can solve the following exercises. If you have difficulties, please review the appropriate prerequisite modules.

(Answers below.)

Recall the formula for slope; the slope of the line joining the points  $(x_1, y_1)$  and  $(x_2, y_2)$  is:

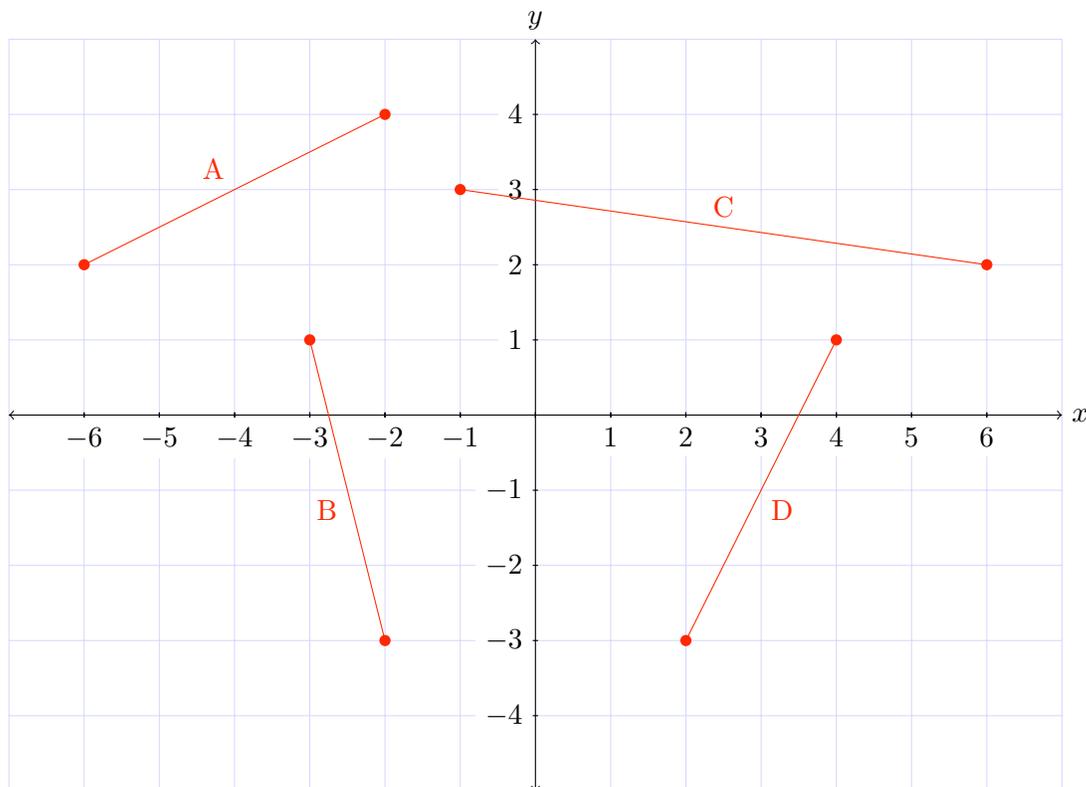
$$\begin{aligned}\text{slope} &= \frac{\text{rise}}{\text{run}} \\ &= \frac{\Delta y}{\Delta x} \\ &= \frac{y_2 - y_1}{x_2 - x_1}\end{aligned}$$

1. Calculate the rise between each pair of points.  
(a)  $A(15, 20)$  and  $B(30, 45)$       (b)  $C(0.5, 0.3)$  and  $D(0.25, 0.1)$   
(c)  $E(-1, -6)$  and  $F(-8, 1)$       (d)  $G(16, -3)$  and  $H(-9, -14)$

2. Calculate the run for each pair of points in Question 1.

3. Using the results of Questions 1 and 2, calculate the slope of a line segment connecting each pair of points.

4. Calculate the slope of each the line segment in the following graph:



5. Determine the slope of each line.

- (a)  $y(x) = 6x + 3$       (b)  $2 - h = p(h)$   
(c)  $2 + m(n) = 16n$       (d)  $2(t + 10) = w(t)$

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Answers: 1.(a) 25 (b)  $-0.20$  (c) 7 (d)  $-11$  2.(a) 15 (b)  $-0.25$  (c)  $-7$  (d)  $-25$  3.(a)  $5/3$  (b)  $4/5$   
(c)  $-1$  (d)  $11/25$  4.(a)  $1/2$  (b)  $-4$  (c)  $-1/7$  (d) 2 5.(a) 6 (b)  $-1$  (c) 16 (d) 2

## Introduction

Rates of change are used to calculate the relationship between two variables, or to observe trends or patterns in a set of data. More specifically, rates of change examine what happens to the dependent variable as the independent variable changes. Rates of change are used daily by individuals working in all job disciplines—from medical researchers trying to determine how effective a particular product is over a specified time span, to government officials interested in analyzing a country's population growth or decline. Using rates of change, automobile mechanics can determine at which speeds an automobile uses the most/least amounts of gasoline with the goal of making vehicles more energy efficient.

In this module we'll study rates of change and how one can determine a rate of change by looking at a graph.

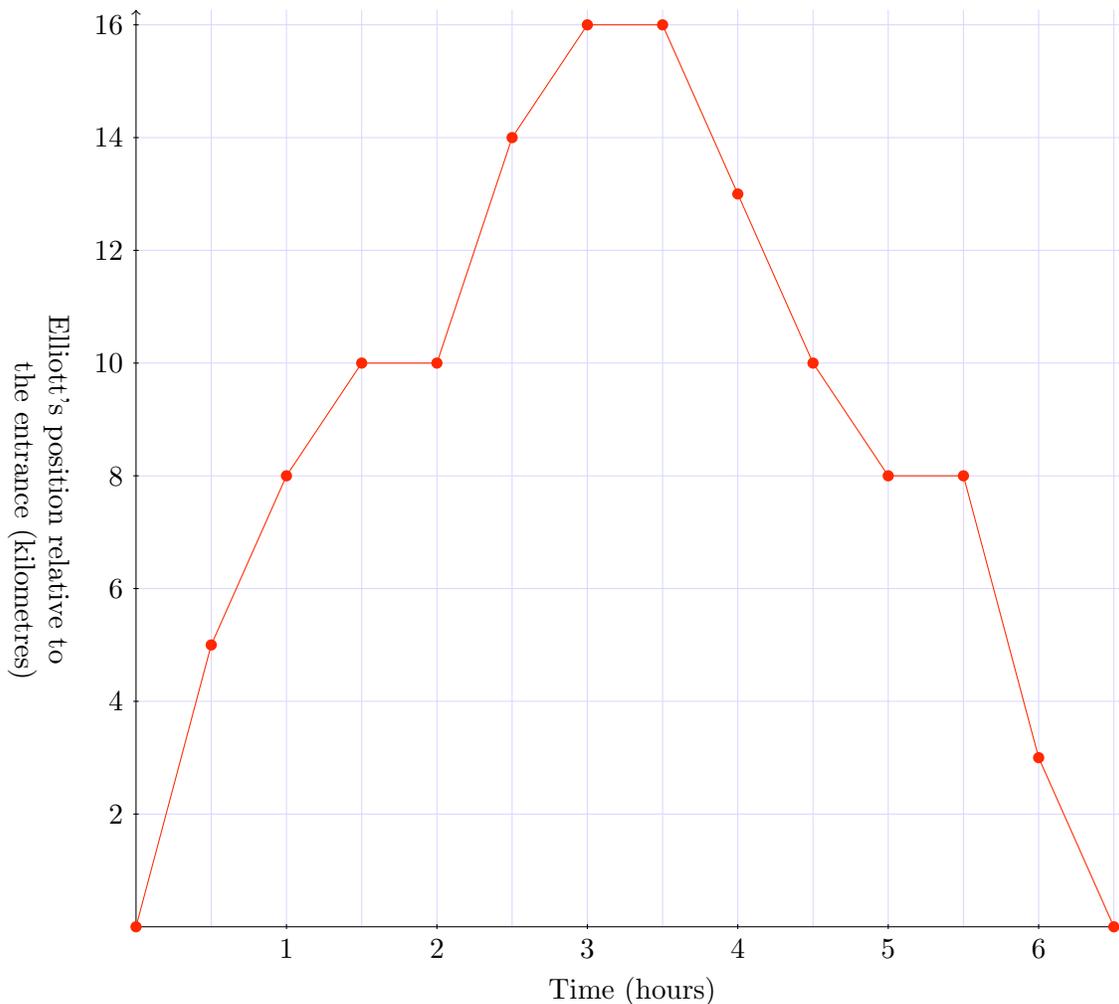
### FOCUS QUESTION

To help you understand an important aspect of this lesson, focus your attention on this question, which will be answered towards the end of the lesson.

Sam's aunt had a baby in February. The baby's initial weight was 3.6 kg. In March the baby grew to a weight of 4.1 kg and by May the baby's weight was 6.4 kg. How fast was the baby's weight changing between February and March? March and May? February and May?

### Rate of change for a motion

Elliott, an avid cyclist, decided to go on a bike ride through a nearby national park. Elliott began cycling once he entered the park. The following graph represents his progress.



Rates of change are used to understand the relationship between two variables. For Elliott's bike ride through the park, we'll investigate the relationship between time and Elliott's position. We imagine that the trail that Elliott is on has markers every so often labelled in kilometres. (Highways have similar markers.) In this way Elliott can keep track of his position just by looking at the marker next to him on the trail.

Note that the  $x$ -coordinate of any point on the graph represents time, and the matching  $y$ -

coordinate represents Elliott's position.

Let's begin at the left-most point on the graph, which has coordinates  $(0, 0)$ . We interpret these coordinates to mean that at time 0 hours, Elliott's position is 0 km. Surely the time on Elliott's watch doesn't read 0, but we imagine that perhaps a friend of Elliott has a stopwatch which he turns on at a certain time when Elliott is next to the 0 km marker. It's more convenient for us to use the stopwatch time to describe Elliott's motion than to use regular clock time.

In the first half hour of Elliott's journey, he evidently cycles 5 km, because the point  $(0, 0)$  is connected to the point  $(0.5, 5)$  on the graph. The latter point means when the stopwatch reads 0.5 h, Elliott's position is 5 km. Note that these two points on the graph are connected with a line segment. As you've learned in the past, the slope of the line segment joining  $(0, 0)$  and  $(0.5, 5)$  is:

$$\begin{aligned}\text{slope} &= \frac{5 - 0}{0.5 - 0} \\ &= 10\end{aligned}$$

What does the value of this slope tell us about Elliott's motion in the first half hour? We can understand this by including the units in the slope calculation:

$$\begin{aligned}\text{slope} &= \frac{5 \text{ km} - 0 \text{ km}}{0.5 \text{ hours} - 0 \text{ hours}} \\ &= \frac{5 \text{ km}}{0.5 \text{ hours}} \\ &= 10 \text{ km/h}\end{aligned}$$

The unit for the slope is km/h (kilometres per hour), which is a velocity unit. Evidently Elliott's velocity for the first half hour of his journey is 10 km/h. Elliott's velocity is a rate of change, because it indicates the rate at which his position is changing with respect to time: his position is increasing at a rate of 10 km per hour. The fact that Elliott's rate of change of position is positive indicates that he is moving in the positive direction; we've implicitly taken this to mean the direction of the path away from the entrance. We'll consider the negative direction to mean in the direction back towards the entrance.

Did Elliott ride at 10 km/h throughout his bike ride, though?

By looking at the graph we can see that the slope of the line changes at the 0.5 hour mark. Just as before, we have a straight line connecting the 0.5 hour mark to the 1 hour mark. As we just learned, this means that there is a *constant rate of change* along this line segment. By constant rate, we mean that the slope remains the same for a certain time interval. From the graph, we can see that there is a constant rate of change between 0 and the 0.5-hour mark, and between the 0.5-hour mark and the 1-hour mark.

Now let's calculate the slope of the line segment between the 0.5-hour mark and the 1-hour mark, including units!

$$\begin{aligned}\text{slope} &= \frac{8 \text{ km} - 5 \text{ km}}{1 \text{ hour} - 0.5 \text{ hours}} \\ \text{velocity} &= \frac{3 \text{ km}}{0.5 \text{ hours}} \\ &= 6 \text{ km/h}\end{aligned}$$

Therefore, we know that between 0.5 hours and 1 hour, Elliott is cycling at a constant velocity of 6 km/h. Since Elliott's speed is positive for this time interval, it means he is travelling away

from the entrance. Thus we can say that Elliott was cycling at a velocity of 6 km/h away from the entrance of the park.

We can also use + and – signs to indicate direction of velocity, where the positive sign indicates motion along the path going farther from the entrance, and the negative sign indicates motion along the path going towards the entrance. Using positive and negative signs will save us from saying “away from the park entrance,” and “towards the park entrance,” over and over again.

Now let’s compare our first two calculations: For the first half hour, Elliott cycles at a velocity of +10 km/h (i.e., away from the park entrance). For the following half hour Elliott cycles at a velocity of +6 km/h. We can see that at the 0.5 hour mark, Elliott’s velocity decreases from +10 km/h to +6 km/h. Perhaps Elliott slowed his pace in the second half hour because he became tired.

Now look at the third line segment, between 1 hour and 1.5 hours; we can again see that a line segment connects the two points, so there will be a constant rate of change of position.

**PRACTICE**

(Answers below.)

1. Calculate Elliott’s velocity in the time interval between 1 hour and 1.5 hours.

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Answers: 1. +4 km/h

Note that Elliott’s velocity in the time interval between 1 hour and 1.5 hour is still positive, so he continues to move away from the park entrance. However, in each subsequent half hour, his velocity is decreasing, so perhaps he is becoming increasingly tired.

Now calculate the rate of change of Elliott’s position for the next half-hour interval.

**PRACTICE**

(Answers below.)

2. Calculate Elliott’s velocity in the time interval between 1.5 hour and 2 hours.

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Answers: 2. 0 km/h

You will have noticed that Elliott’s velocity in the time interval between 1.5 hours and 2 hours is 0. Another way to say this is that the rate of change of his position is 0; in other words, Elliott’s position does *not* change in the time interval between 1.5 hours and 2 hours. Perhaps Elliott decided to take a lunch break! You can verify from the graph that during this time interval Elliott’s position is a constant 10 km from the park entrance, so he does not move in this interval.

## PRACTICE

(Answers below.)

3. Calculate Elliott's velocity for each time interval.

(a) from 2 h to 2.5 h

(b) from 2.5 h to 3 h

(c) from 3 h to 3.5 h

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Answers: 3.(a) +8 km/h (b) +4 km/h (c) 0 km/h

Continuing with Elliott's graph, we can see that between the 2 hour and 2.5 hour mark, Elliott again cycles at a constant rate of +8 km/h, and in the subsequent half hour, Elliott slows down to a rate of +4 km/h. This suggests that Elliott, after his lunch break, began cycling fast for the first half hour, and then became tired once again and slowed down.

During the 3 to 3.5 hour interval, we can see that once again Elliott's rate of change is 0 km/h, which implies that he took another break.

Let's move on and look at the rest of Elliott's bike ride!

Notice that the character of Elliott's position-time graph changes at the 3.5 hour mark. During the 3.5 h to 4.5 h interval, the line slopes downwards. In the previous time intervals, the line segments either sloped upwards or were level. Let's calculate the rate of change of Elliott's position during the 3.5 h to 4.5 h time interval:

$$\begin{aligned}\text{velocity} &= \frac{10 \text{ km} - 16 \text{ km}}{4.5 \text{ hours} - 3.5 \text{ hours}} \\ &= \frac{-6 \text{ km}}{1 \text{ hour}} \\ &= -6 \text{ km/h}\end{aligned}$$

Hold on! How can Elliott be cycling at a negative velocity?? In the previous calculations, all of Elliott's rates were positive, which we said means that Elliott was cycling *away* from the park entrance. When the rate of change is negative, as in this case, it means that Elliott is cycling *towards* the park entrance.

Therefore, we can conclude that between 3.5 hours and 4.5 hours, Elliott is cycling at a constant rate of 6 km/h *towards* the park entrance. In other words, after Elliott took his second break, he decided to turn around and head back to the entrance of the park, possibly because he had reached the end of the bike path and was forced to turn around and head back.

Now what happened between the 4.5 hour and 5 hour marks? Let's calculate Elliott's velocity:

$$\begin{aligned}\text{velocity} &= \frac{8 \text{ km} - 10 \text{ km}}{5.0 \text{ hours} - 4.5 \text{ hours}} \\ &= -4 \text{ km/h}\end{aligned}$$

We can thus see that Elliott began cycling fast towards the park entrance, and once again became tired and slowed his rate of change of position to 4 km/h.

If we were to finish calculating Elliott's velocity through the park, we would see:

- Between the 5 hour and 5.5 hour marks, Elliott took a third break, and did not move. Perhaps he found a beautiful outlook in the park and decided to take in the sights!
- Between the 5.5 hour and 6 hour marks, Elliott's velocity is  $-10$  km/h, which means his rate of change of position was 10 km/h towards the entrance of the park. Elliott's break must have given him back a great deal of energy, because during this leg of his journey, he was cycling at a high rate.
- Finally, between the 6 hour and 6.5 hour marks, Elliott's rate of change of position slowed to 6 km/h towards the entrance. By looking at the graph, we can also see that at 6.5 hours, Elliott was 0 km from the entrance of the park; this means that Elliott returned to the entrance of the park after 6.5 hours of cycling within the park.

What did we learn from this example?

- When determining a rate of change, first understand the relevant variables. It is important to understand the real world applications of your rates of change. This will also help you interpret both positive and negative rates.
- It is not only important to analyze the slope of the function at hand, but it is also important to study the graph to gain information. For instance, by solely looking at the graph, we can see that by the 6.5 hour mark, Elliott had returned to the entrance of the graph, because his distance from the entrance was at 0 km.
- We learned how to calculate a rate of change—in this case, Elliott's velocity—by interpreting a graph.

### KEY IDEA

Whether a rate of change is considered positive or negative is sometimes based on perspective. That is, we can sometimes choose to redefine variables to make rates come out with one sign or the other.

For instance, if we were to change all of the kilometre markers in the park so that they numbered 0 at the end of the trail, and increased to 16 km at the beginning of the trail, then the signs of Elliott's velocities would all be reversed. Try it yourself: what would the graph look like if we were to adopt this convention?

In many applied problems, we have some freedom in choosing variables (or equivalently, choosing a coordinate system). Choosing wisely is an important skill for those who construct and apply mathematical models in applications; as you grow in your understanding, you will develop this skill further.

The next example deals with a rate of change that is similar to the previous one, in that it also involves time, but in this case we don't usually speak of velocity.

## EXAMPLE 1

For a school project, Kimberly sets out a rain barrel in her backyard to monitor rainfall. On the first day there was no rain in the barrel. By the second day, there was 2 cm of water in the barrel. By day three, the water level was at 1 cm. On day four, Kimberly was surprised to find 5 cm of water, and on the final day, day five, Kimberly recorded the amount of water to be at 2 cm.

How fast was the water level changing between (a) day 1 and day 2? (b) day 2 and day 3? (c) day 2 and day 5?

## SOLUTION

First, let's decide which two variables we will use. We would like to determine how the water level in the rain barrel changes over time; therefore our two variables are time (measured in days) and water level (in centimetres).

Next we must determine which direction will be positive and which will be negative for the amount of rainwater. If we imagine putting a metre-stick into the rain barrel, so that the end marked 0 is at the bottom of the barrel, then the amount of rainwater in the barrel is either 0 or a positive number, and the variable we use to indicate the level increases as the amount of water in the barrel increases.

(a) Now we can move forward and answer the first part of the question. We must determine the rate of change of the water level between day 1 and day 2. In order to solve this problem we can use the same slope formula as we used for Elliott's bicycle ride:

$$\begin{aligned}\text{rate of change of water level} &= \frac{2 \text{ cm} - 0 \text{ cm}}{2 \text{ days} - 1 \text{ day}} \\ &= 2 \text{ cm/day}\end{aligned}$$

Therefore between day 1 and day 2, the water level increased at a rate of 2 cm per day. Perhaps there was a light rain overnight, which could explain why there was now water in the barrel.

(b) Let's look at the rate of change for days 2 and 3:

$$\begin{aligned}\text{rate of change of water level} &= \frac{1 \text{ cm} - 2 \text{ cm}}{3 \text{ days} - 2 \text{ days}} \\ &= -1 \text{ cm/day}\end{aligned}$$

The negative rate of change means that between day 2 and day 3, the amount of water in the barrel *decreased* at a rate of 1 cm/day. (Maybe it was a hot summer day and the water evaporated from the barrel.) Just as in Elliot's bicycle ride, a water level moving in one direction (up) is described by a positive rate of change, and a water level moving in the opposite direction (down) is described by a negative rate of change.

(c) Finally, the rate of change of water in the barrel between day 2 and day 5 is:

$$\begin{aligned}\text{rate of change of water level} &= \frac{2 \text{ cm} - 2 \text{ cm}}{5 \text{ days} - 2 \text{ days}} \\ &= 0 \text{ cm/day}\end{aligned}$$

Thus, the rate of change of the water level between day 2 and day 5 is 0 cm/day. This may seem puzzling at first, because there were changes in the water level during the five days. However, the rate of change of water level between day 2 and day 5 represents the *average* rate of change of the water level during that time interval; another way to say this is the *net* rate of change. Because the starting and ending water levels were the same, the net rate of change of water level is zero, even though there were changes in water level during the time interval.

## INVESTIGATION

Explore this on your own!

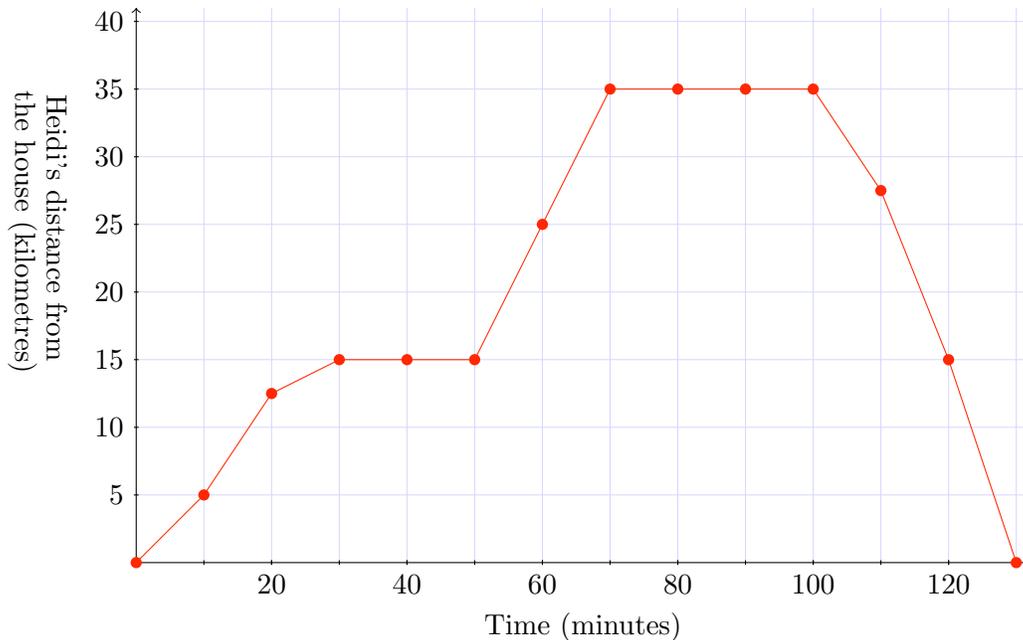
Try this at home: Set up a cylindrical bucket or bowl outside, and let it sit for approximately 5 days. Measure the amount of water in the bowl at the same time each day, and record the amounts. Then calculate rates of change of water level for various time intervals, just as Kimberly did. You might like to present your results using graphs or tables.

Now it's your turn: Put what you have learned to the test with some questions that you can try yourself:

## PRACTICE

(Answers below.)

4. Heidi and her mom have decided to go shopping at two different stores. They left their house at 4:00 pm. Their shopping trip is displayed in the graph below, where the  $x$ -axis represents time, and the  $y$ -axis represents their distance away from their house.

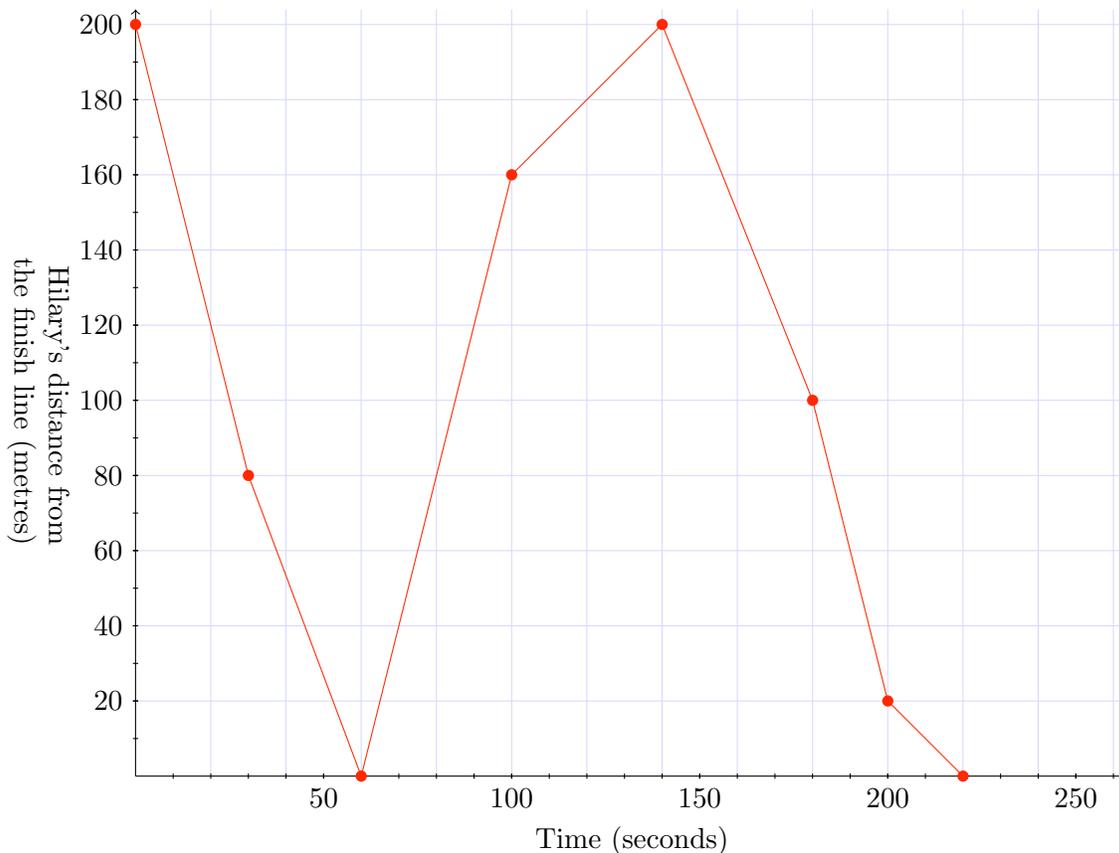


- (a) At the 30 minute mark, how fast were Heidi and her mom driving?
- (b) When were Heidi and her mom driving the fastest? Explain your reasoning.
- (c) Not including the times when the vehicle was stopped, when were Heidi and her mom driving the slowest? Explain your reasoning.
- (d) After how many minutes of driving did Heidi and her mom reach the first store? Explain your reasoning.
- (e) After how many minutes of driving and shopping did Heidi and her mom reach the second store?
- (f) At what time did Heidi and her mom return home?
5. Colin is getting ready to go on summer vacation in a couple days and has been paying close attention to the temperatures for one week in order to determine what type of clothing to pack. On Sunday, the temperature was  $20^{\circ}$  Celsius. On Monday, the temperature changed to  $24^{\circ}$  C, and by Tuesday it was  $28^{\circ}$  C. By Wednesday the temperature was  $21^{\circ}$  C. On Thursday the temperature was  $15^{\circ}$  C, and once again on Friday, the temperature was  $20^{\circ}$  C. Finally, on Saturday the temperature was  $25^{\circ}$  C.
- (a) What was the rate of change of the temperature between Sunday and Monday?
- (b) What was the rate of change of the temperature between Monday and Tuesday?
- (c) What was the rate of change between Sunday and Saturday?
- (d) Between which two consecutive days was the rate of change in temperature the greatest? What was the rate of change in temperature?
- (e) Based on the results of Parts (a) to (d), should Colin expect consistent temperatures during his vacation?
6. Henry purchased a 2001 Nash Dribbler automobile for \$19 300. The following table shows how the value of the car changed during the following decade.

Year	Value (in Dollars)
2001	19 300
2002	18 500
2003	18 000
2004	17 750
2005	17 400
2006	17 000
2007	16 510
2008	15 900
2009	15 525
2010	14 950

- (a) What was the rate of change in value between the year 2001 and the year 2010?
- (b) What is the rate of change in value of the car between 2008 and 2010?
- (c) Was the rate of change in value between the years 2002 and 2007 *greater than* or *less than* the rate of change in value between the years 2003 and 2008?
- (d) Between which two consecutive years did the car depreciate in value the least? What was the rate of change of the car's value between these two years?

7. Hilary is training for a 600 m race. Normally she trains on an oval track, but it's under construction, so she trains by running back and forth in a parking lot. The parking lot is 200 m long. Hilary's coach uses a stopwatch and records the times for one of her training runs; the data is summarized in the graph below.



- How long did it take Hilary to run the first 200 m?
- How long did it take Hilary to complete the entire training run?
- What was Hilary's fastest speed, and for how long did she run at this pace?
- What was Hilary's slowest speed? Speculate as to why her speed was so low at this point in the race.
- Compare Hilary's pace between the 100–140 second mark with her speed from the 140–180 second mark.

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Answers: 4. (a) 1 m/min.

- (b) From 120-130 minutes. This interval has the largest rate of change.  
 (c) From 20-30 minutes. This interval has the smallest rate of change that is  $> 0$  km/min.  
 (d) 30 minutes. This is the first point at which the rate of change is zero.  
 (e) 70 minutes.  
 (f) 7:10 p.m.

5. (a) 4 degrees/day.  
 (b) 4 degrees/day.  
 (c) 0.833° /day  
 (d) Tuesday to Wednesday. Temperature decrease of 7°/day.  
 (e) The rates of change were between 4 and 7 degrees per day- both increasing and decreasing, which is a measurable change in temperature, therefore Colin should not expect consistent temperatures.
6. (a) Decreased in value by 483.33 dollars/year approximately.  
 (b) 475 dollars/year.  
 (c) Less than.  
 (d) Between 2003 and 2004. 250 dollars/year.
7. (a) 60 seconds.  
 (b) 220 seconds.  
 (c) Hilary ran at a pace of 4 m/s between: 0 and 30 seconds, 60 to 100 seconds, and 180 to 200 seconds; thus she ran at this pace for a total of 90 seconds.  
 (d) Hilary's slowest pace was 1 m/s. She ran at this pace twice- around the half way mark of the race and during the final 20 seconds of the race. Perhaps she ran too fast in the beginning and by the half way mark, had to slow down in order to regain some energy. By the end of the race, Hilary likely had no fuel left in the tank and was tiring fast which resulted in her slower speed.  
 (e) For 100-140 seconds into the race, Hilary was running at a velocity of 1 m/s AWAY from the finish line. For the next 40 seconds, Hilary ran at a velocity of 2.5 m/s TOWARDS the finish line. At 140 seconds, Hilary reached the end of the parking lot, which resulted in this change of direction.

### RECAP OF FOCUS QUESTION

Recall the focus question, which was asked earlier in the lesson.

**Sam's aunt just had a baby in February. The baby's initial weight was 3.6 kg. In March the baby had grown to a weight of 4.1 kg and by May the baby's weight was 6.4 kg. How fast was the baby's weight changing between February and March? March and May? February and May?**

### SOLUTION

The rate of change of the baby's weight between February and March is

$$\begin{aligned} \text{rate of change of weight} &= \frac{4.1 \text{ kg} - 3.6 \text{ kg}}{1 \text{ month}} \\ &= 0.5 \text{ kg/month} \end{aligned}$$

The rate of change of the baby's weight between March and May is

$$\begin{aligned} \text{rate of change of weight} &= \frac{6.4 \text{ kg} - 4.1 \text{ kg}}{2 \text{ month}} \\ &= 1.15 \text{ kg/month} \end{aligned}$$

The rate of change of the baby's weight between February and May is

$$\begin{aligned} \text{rate of change of weight} &= \frac{6.4 \text{ kg} - 3.6 \text{ kg}}{3 \text{ month}} \\ &= 0.933 \text{ kg/month} \end{aligned}$$

## INVESTIGATION

Explore this on your own!

Using the information gathered from the Focus Question, try to estimate what would happen in the future if the baby maintained the same weight gain. Estimate what the baby's weight would be in August and September.

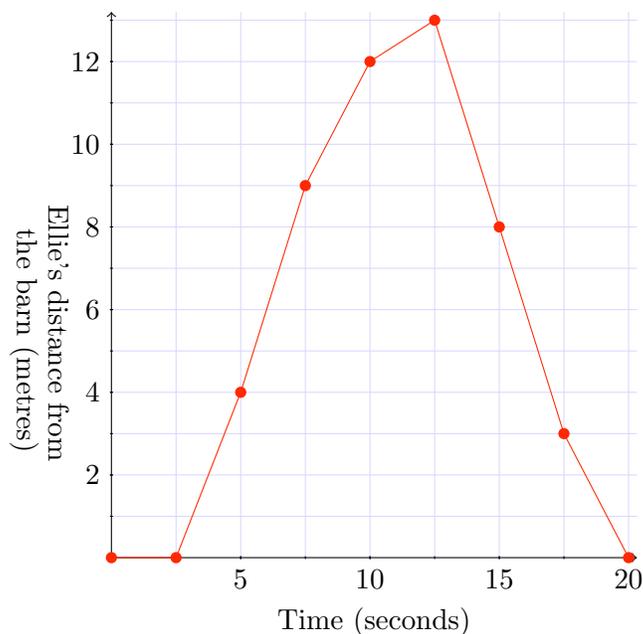
You might like to look up growth charts for infants online. What are the typical times for which an infant's growth is the greatest? How does growth in each month compare for the first year? How does growth in each year compare for the first few years?

### WWW

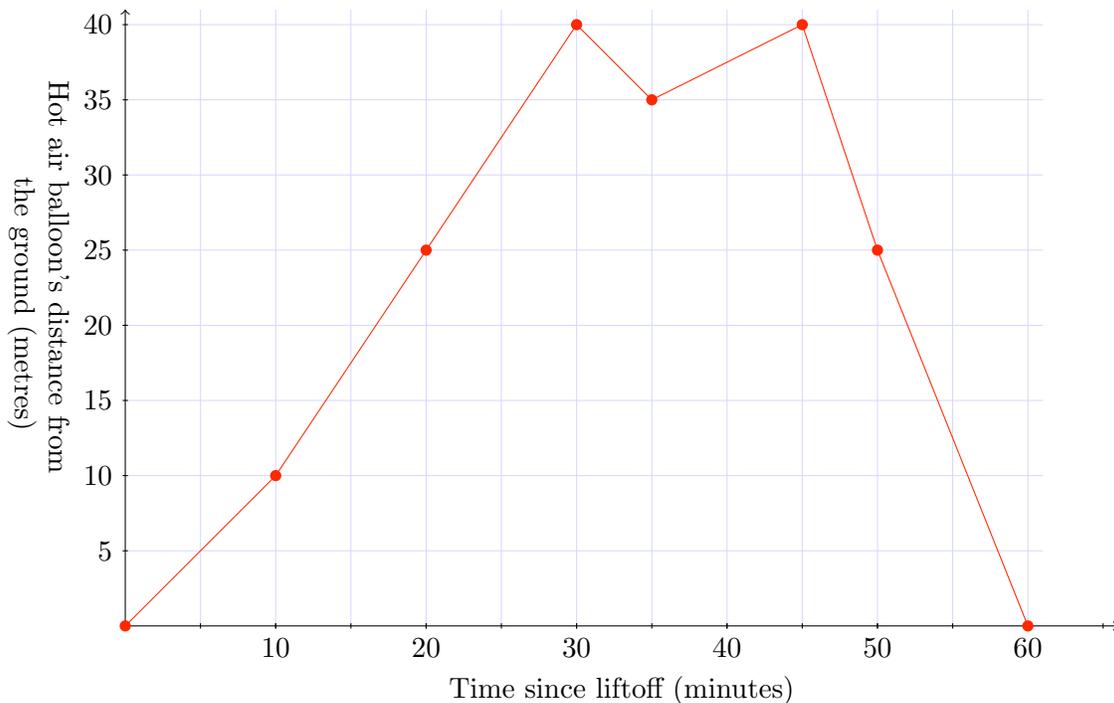
- What we did: We learned how to calculate rates of change and how to interpret them in realistic contexts.
- Why we did it: Rates of change are useful for understanding the relationship between two variables.
- What's next: Learning to calculate *instantaneous* rates of change, which are more accurate than average rates of change (as calculated in this module) for situations where the rate of change varies continuously.

### EXERCISES

1. Ellie goes horseback riding for her birthday. Ellie starts her stopwatch the moment she arrives at the barn. The graph describes her position for her ride.



- (a) After how many seconds did Ellie leave the barn?
  - (b) What was Ellie's fastest pace riding away from the barn?
  - (c) What was Ellie's slowest pace riding towards the barn?
  - (d) After how many seconds did Ellie change directions?
  - (e) Did Ellie ever ride at a constant rate for at least 5 seconds? If so, during which time interval?
2. Since Patrick was young, he measured his height and marked it off on a piece of wood. When Patrick was born, he measured 1.8 feet tall. When Patrick was 2 years old, his height was 2.8 feet tall. By the time he was 4, he had grown to be 3.2 feet tall. On his 6th birthday, Patrick was 3.8 feet tall. Today, at the age of 8, Patrick is 4.3 feet tall.
- (a) How fast was Patrick's height changing between 6 and 8 years?
  - (b) Did Patrick ever grow at a constant rate over the span of 4 consecutive years?
  - (c) During which 2 year span was Patrick growing the fastest? (ie: 0–2 years, 2–4 years, 4–6 years, 6–8 years)
  - (d) During which 4 year span was Patrick growing the slowest? (ie: 0–4 years, 2–6 years, 4–8 years)
  - (e) If Patrick continued to grow at the same pace as he did between 6 and 8 years of age, how tall can we expect Patrick to be at the age of 10?
3. Joanne goes hot air ballooning on a beautiful summer evening. The balloon company plotted the balloon's height in the following graph.



- (a) How long was the balloon in the air?
  - (b) When was the balloon ascending?
  - (c) When was the balloon descending?
  - (d) Did the balloon travel at a constant rate for longer than 15 minutes at any point in the ride? If so, when?
  - (e) What was the balloon's fastest pace travelling *away* from the ground?
  - (f) What was the balloon's slowest pace travelling *towards* the ground?
  - (g) What was the balloon's highest altitude from the ground?
4. Quinn would like to go skiing at a nearby ski club, but the amount of snow on the hill continues to change. When the snow base is less than 15 cm thick, the club will not allow skiers on the hill. For one week Quinn has decided to record the snow base levels on the hill in the mornings. Here are his results:

Day	Snow Level
Sunday	20 cm
Monday	35 cm
Tuesday	40 cm
Wednesday	48 cm
Thursday	37 cm
Friday	30 cm
Saturday	27 cm

- (a) Between which two consecutive days was the change in snow level the greatest?
- (b) What was the rate of change in snow level from Sunday to Wednesday?
- (c) What was the rate of change in snow level from Wednesday to Saturday?
- (d) Did the snow level ever change at a constant rate over the course of 3 consecutive days during Quinn's test?
- (e) If the snow level continues to change at the same rate as it did from Friday to Saturday, when should Quinn expect the ski hill to be closed?