BROCK UNIVERSITY MATHEMATICS MODULES

12B1.3: Soh Cah Toa

Author: Tyson Moss

WWW

- What it is: SohCahToa is a memory aid for remembering the definitions of the sine, cosine, and tangent functions for an angle that can fit into a right triangle.
- Why you need it: Knowing the definitions of the basic trigonometric functions allows you to solve all kinds of geometric problems.
- When to use it: Solving geometric problems, especially determining an unknown side or angle of a right triangle, and also solving related applied problems.

PREREQUISITES

Before you tackle this module, make sure you have completed these modules:

Degree Measure, Introduction to Radian Measure 12B1.1, Similar Triangles, Pythagoras's Theorem, Unit Conversions

WARMUP

Before you tackle this module, make sure you can solve the following exercises. If you have difficulties, please review the appropriate prerequisite modules.

(Answers below.)

1. Determine the third angle of a triangle, given the first two.

   (a) $A = 60°$; $B = 30°$ (b) $A = 50°$; $B = 50°$

2. Determine the fourth angle of a quadrilateral, given the first three.

   (a) $A = 60°$; $B = 30°$; $C = 90°$ (b) $A = 167°$; $B = 32°$; $C = 83°$

3. Triangle ΔABC has sides of length $AB = 5$, $BC = 6$, and $AC = 7$. Triangle ΔDEF is similar to triangle ΔABC. The length of DE is 8. Calculate the lengths of the other two sides of triangle ΔDEF.

4. Determine the unknown side length of each right triangle, where c represents the length of the hypotenuse and a and b represent the lengths of the other sides.

   (a) $a = 5$, $b = 6$ (b) $a = 4$, $c = 9$

5. Convert each length into the given unit.

   (a) 6 cm to mm   (b) 15 mm to m   (c) 2.5 km to cm

Answers: 1.(a) 90 (b) 80 2.(a) 180 (b) 78 3. $EF = 9.6$, $DF = 11.2$ 4.(a) $c = \sqrt{61}$ (b) $b = \sqrt{65}$ 6.(a) 60 mm (b) 0.015 m (c) 250 000 cm

Introduction

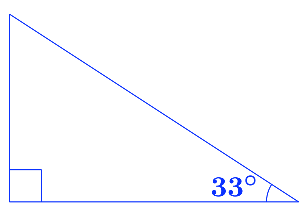
Soh Cah Toa is a memory aid that helps us remember the definitions of the trigonometric functions sine, cosine, and tangent. Trigonometric functions occur often in mathematical models of nature, especially in describing repetitive, back-and-forth, or periodic motions, such as vibrations, waves, merry-go-rounds, your heart beating, etc. And trigonometric functions are also very helpful in solving a wide variety of geometric problems, especially those involving triangles.

In this module you'll learn how the basic trigonometric functions (sine, cosine, and tangent) are defined for angles that fit inside a right triangle;[1] that is, for angles that are greater than 0°, but less than 90°. In Module ***, you'll learn to generalize these basic definitions to angles of any size.

FOCUS QUESTION

To help you understand an important aspect of this lesson, focus your attention on this question, which will be answered towards the end of the lesson.

Determine the unlabelled angles and sides in the following triangle. Then write expressions for the sine, cosine, and tangent of the two non-right angles in terms of the sides of the triangles.



Soh Cah Toa is a memory aid used to remember the definitions of the trigonometric functions as defined within right angled triangles. The acronym stands for sine - opposite - hypotenuse; cosine - adjacent - hypotenuse; tangent - opposite - adjacent. Let's explain each one in turn.

Soh

Soh stands for *sine - opposite - hypotenuse*. If you were to read it aloud, you would say *sine is opposite over hypotenuse*. This defines the sine of an angle within a right triangle in terms of the sides of the triangle. For example, suppose you want to calculate the sine of angle θ in the Figure 1.

[1]In the past these were commonly known as "right-angled triangles," but nowadays "right triangle" is more common.

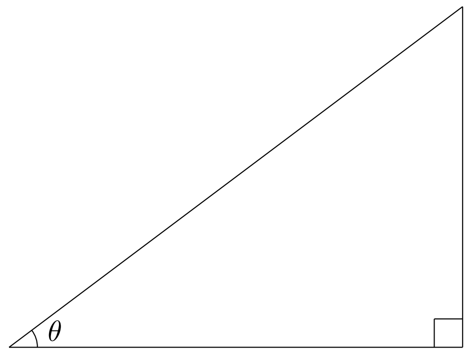


Figure 1: A right triangle including angle θ.

It's conventional to label the sides of the right triangle as follows: the *hypotenuse* is labelled h, the side of the triangle opposite to angle θ is labelled with o, and the side adjacent to angle θ is labelled with a. See Figure 2.

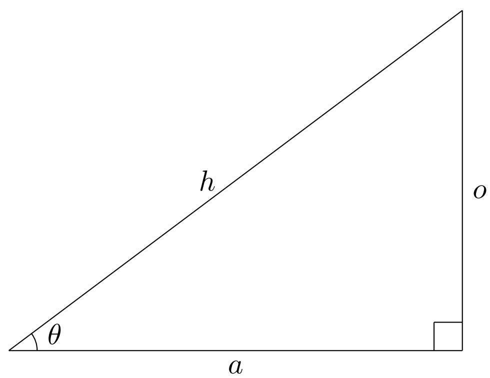


Figure 2: The sides of the right triangle are labeled with respect to angle θ.

The acronym *Soh* is meant to remind us of the following definition:

DEFINITION

Suppose that θ is an angle within a right triangle. Then the sine of angle θ is the length of the opposite side o divided by the length of the hypotenuse h. In other words:

$$\sin(\theta) = \frac{o}{h}$$

Using this definition, you can calculate the sine of each of the non-right angles in a right triangle if you know the lengths of the sides.

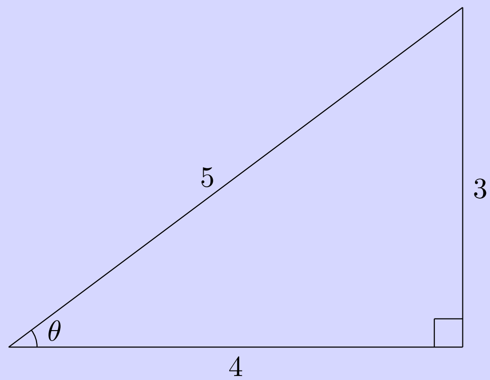
Now hold on a minute, some of you may be saying. *What about right triangles that have the same angles but the sides are completely different? Surely you must be mistaken!*

Well, that's an interesting point. Triangles with the same angles are called *similar triangles*. And you may recall from high school that for similar triangles, corresponding sides are in the same ratio. This means that if you calculate the ratio of opposite to hypotenuse for one of the triangles, and then calculate the ratio of opposite to hypotenuse for the similar triangle, the ratios will be the same. This means that the sine of the angle depends only on the angle, and will be the same no matter which right triangle you use for the calculation.

Let's look at an example.

EXAMPLE 1

Given the following triangle, calculate $\sin(\theta)$ without the use of a calculator.



SOLUTION

First identify the sides of the right triangle. The hypotenuse is the longest side of the triangle, opposite the right angle. Thus, $h = 5$. The other side of the triangle that is closest to angle θ is the adjacent side, so $a = 4$. Finally, the remaining side is opposite the angle θ, so $o = 3$.

Using the definition of the sine,

$$\begin{aligned} \sin(\theta) &= \frac{o}{h} \\ &= \frac{3}{5} \\ &= 0.6 \end{aligned}$$

If the sides of the triangle were all increased, but in a way that keeps the angles the same, then all sides would increase by the same relative (i.e., percentage) amount. This means that the sides o and h would increase by the same multiple, which would keep their ratio the same. Try it yourself with a few different similar triangles; for example, you could double all the sides, triple them, multiply all sides by a factor of 1.7, etc., and the sine of the angle θ would be the same for all of the similar triangles.

Cah

Cah stands for *cosine - adjacent - hypotenuse*. If you were to read this aloud, you'd say *cosine is adjacent over hypotenuse*. This definition of the cosine can be stated more formally as follows:

DEFINITION

The cosine of angle θ is the length of the adjacent side a divided by the length of the hypotenuse h. In other words:

$$\cos(\theta) = \frac{a}{h}$$

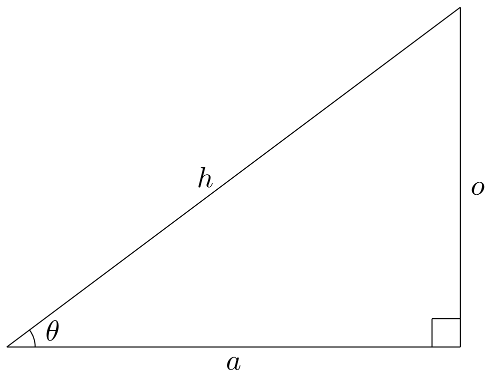
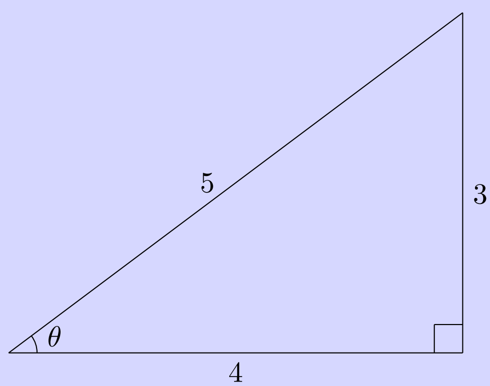


Figure 3: Sides are labeled with respect to angle θ.

Here's an example showing how to calculate the cosine of an angle, using the same triangle as in the previous example.

EXAMPLE 2

Given the following triangle, calculate $\cos(\theta)$ without using a calculator.



SOLUTION

$$\begin{aligned}\cos(\theta) &= \frac{a}{h} \\ &= \frac{4}{5} \\ &= 0.8\end{aligned}$$

Toa

This stands for *tangent - opposite - adjacent*. If you were to read this aloud, you'd say *tangent is opposite over adjacent*. The definition can be stated more formally as follows:

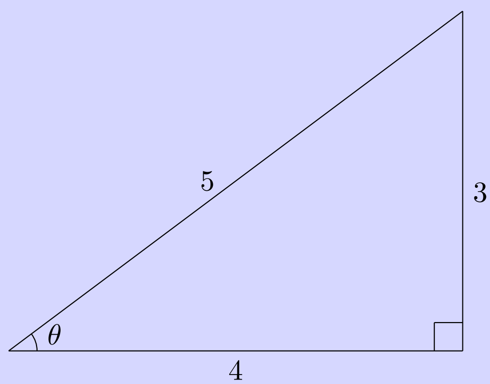
DEFINITION

The tangent of angle θ is the length of the opposite side o divided by the length of the adjacent side a. In other words:

$$\tan(\theta) = \frac{o}{a}$$

EXAMPLE 3

Given the following triangle, calculate $\tan(\theta)$ without the use of a calculator.



SOLUTION

Since this is the same triangle as in the previous two examples,

$$\begin{aligned}\tan(\theta) &= \frac{o}{a}\\ &= \frac{3}{4}\\ &= 0.75\end{aligned}$$

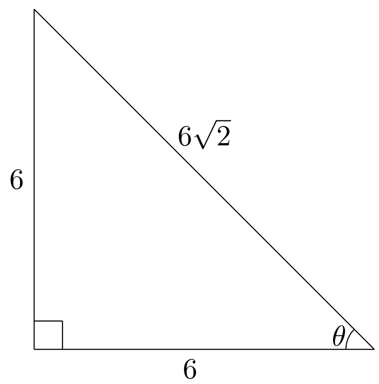
Now use the definitions of sine, cosine, and tangent in a few practice exercises:

PRACTICE

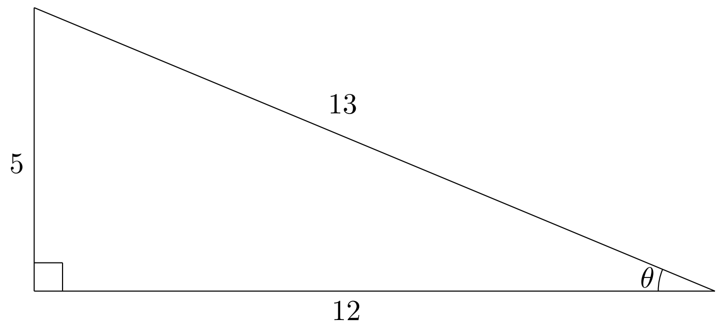
(Answers below.)

Calculate the values of sine, cosine, and tangent for the specified angle θ of each triangle.

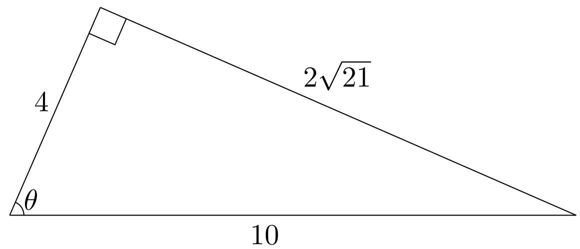
1.



2.



3.



Answers: 1. $\sin(\theta) = \dfrac{1}{\sqrt{2}}$, $\cos(\theta) = \dfrac{1}{\sqrt{2}}$, $\tan(\theta) = 1$

2. $\sin(\theta) = \dfrac{5}{13}$, $\cos(\theta) = \dfrac{12}{13}$, $\tan(\theta) = \dfrac{5}{12}$

3. $\sin(\theta) = \dfrac{\sqrt{21}}{5}$, $\cos(\theta) = \dfrac{2}{5}$, $\tan(\theta) = \dfrac{\sqrt{21}}{2}$

Now, I can hear a few of you already. *When will I EVER use this?* Well, the answer is simple

. . . and to explain it, we'll look at an example. But first, read this key idea!

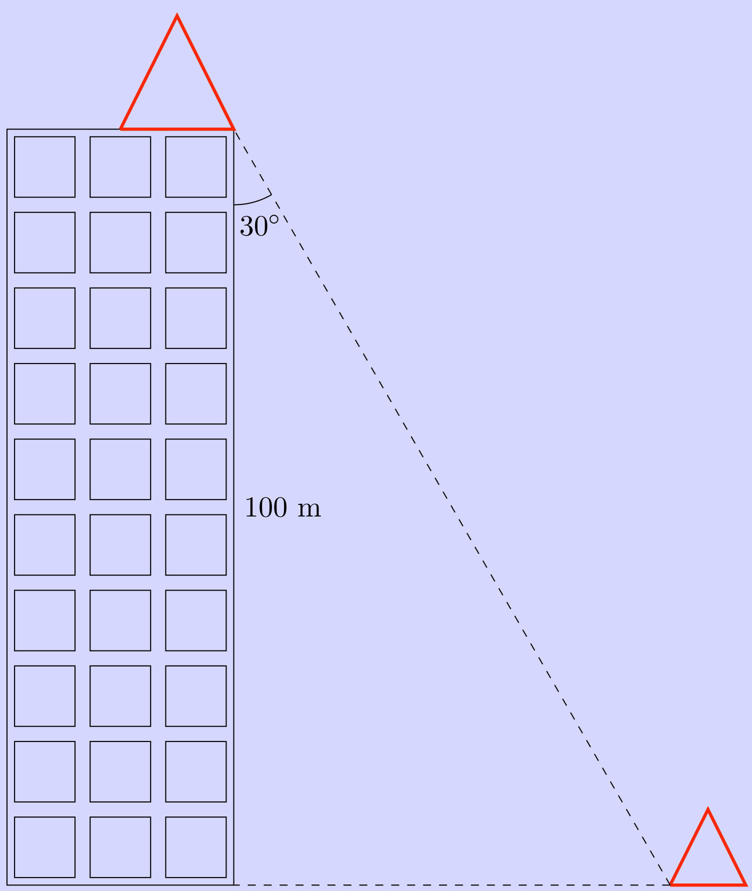
KEY IDEA

The rules of *Soh Cah Toa* aren't only for calculating the angles; they can also be used to find the lengths of the sides of a right triangle. You can even do this if you're only given one angle and one side. The following example will show you how.

EXAMPLE 4

The legendary triangular birds can chirp at frequencies that only their siblings can hear. Another fantastic ability of these legendary birds is that they can immediately tell the angle between two lines just by inspection.

Two of these extraordinary creatures are in a large city with tall buildings. The mother bird is standing in its nest on the top of a building that is 100 m tall, calling for its baby bird, who is at an angle of 30°. How far apart are the baby bird and the mother bird?



SOLUTION

The triangle formed by the dashed lines is a right triangle, which means we are able to apply what we know about *Soh Cah Toa*.

We need to determine the distance between the two red birds, which is the hypotenuse of the dashed triangle. We're also given the height of the building, which is the adjacent side of the dashed triangle.

So, we know an angle in a right triangle, we know the side adjacent to the angle, and we need the hypotenuse. Which ratio do you think we should use? Any of them could be used, but we'll use *Cah*:

$$\begin{aligned} \cos(\theta) &= \frac{a}{h} \\ \cos(30°) &= \frac{100}{h} \end{aligned}$$

Now solve for h to determine the length of the hypotenuse. Doing this, we get:

$$\begin{aligned} \cos(30°) &= \frac{100}{h} \\ h &= \frac{100}{\cos(30°)} \\ h &= \frac{100}{\sqrt{3}/2} \\ h &= \frac{200}{\sqrt{3}} \\ h &\approx 346.4\,\mathrm{m} \end{aligned}$$

This means that the baby bird is about 346 m away from the mother bird.

MAKING CONNECTIONS

The example of the birds may be whimsical, but the same idea allows us to calculate the distances from the earth to stars outside our solar system. Surveyors use the same ideas to calculate inaccessible distances (across rivers when planning for bridges, heights of mountains, etc.).

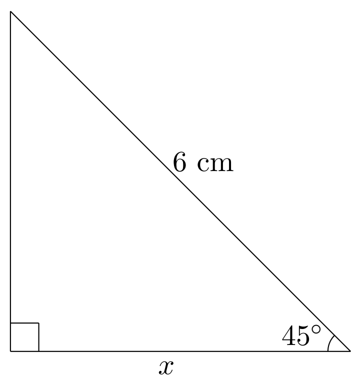
Now it's time to try a few of these problems on your own. Just remember, sometimes the triangle won't be given to you, so be prepared to draw it (it doesn't have to be exact, as long as it correctly guides your thinking).

PRACTICE

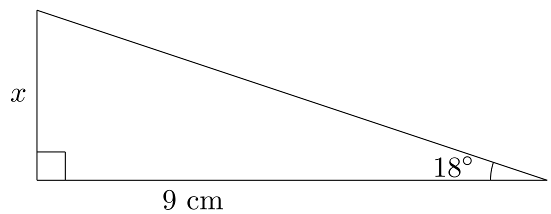
(Answers below.)

4. Calculate the value of x for each triangle.

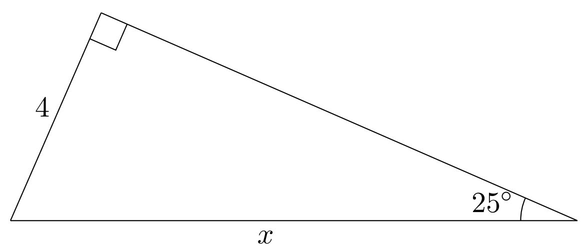
i)



ii)



iii)



5. A sky-diver in a wingsuit (http://en.wikipedia.org/wiki/Wingsuit_flying) jumps out of a plane 4 000 metres above the ground. Wind conditions and the diver's skill allow her to travel in a straight line at an angle of 18° from the vertical.

(a) How far does she travel before landing?

(b) How far away does she land from a point directly below the point where she jumped?

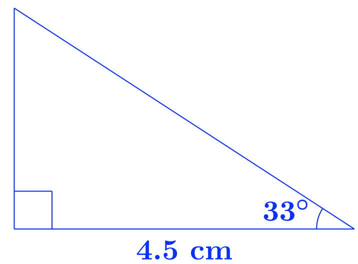
Answers: 4. i) 4.24 cm; ii) 2.92 cm; iii) 9.46 cm; 5. (a) 4205.85 m; (b) 1299.68 m

Now that you have all the tools you need, let's work on that focus question. It should be a piece of cake by now!

RECAP OF FOCUS QUESTION

Recall the focus question, which was asked earlier in the lesson.

Determine the unlabelled angles and sides in the following triangle. Then write expressions for the sine, cosine, and tangent of the two non-right angles in terms of the sides of the triangles.



SOLUTION

Since we know that the interior angles of a triangle add up to 180°, we can determine the angle θ by subtracting the first two angles from 180°:

$$\theta = 180^\circ - 90^\circ - 33^\circ$$
$$\theta = 57^\circ$$

Now that we have all the angles, we need to calculate all the sides. Let's work with the original angle of 33°.

We need to find the hypotenuse and the opposite side, h and o. Let's use the tangent ratio (Toa) to determine the side of the triangle opposite to the 33° angle:

$$\tan(\theta) = \frac{o}{a}$$
$$\tan(33^\circ) = \frac{o}{4.5}$$
$$o = 4.5\tan(33^\circ)$$
$$o \approx 2.9223 \text{ cm}$$

We can use the cosine ratio to determine the hypotenuse:

$$\cos(\theta) = \frac{a}{h}$$
$$\cos(33^\circ) = \frac{4.5}{h}$$
$$h = \frac{4.5}{\cos(33^\circ)}$$
$$h \approx 5.3656 \text{ cm}$$

We can use the *Pythagorean Theorem*[a] as a check on our calculations:

$h^2 = o^2 + a^2$

$a^2 = h^2 - o^2$

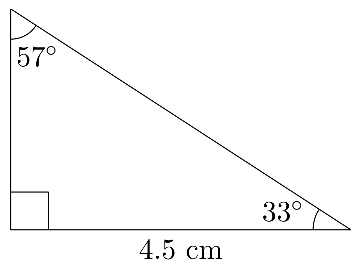
$a = \sqrt{h^2 - o^2}$ (We only use the positive square root, because we are dealing with a length.)

$a = \sqrt{5.3656^2 - 2.9223^2}$

$a = \sqrt{20.249826}$

$a \approx 4.49998$ cm

The checked result for a is extremely close to the given value of 4.5 cm, which gives us confidence in our results. (The very small discrepancy is due to rounding errors.)



So from this, we have all the sides and angles of our triangle. And this is where I'll leave you. You can calculate the trigonometric ratios of all the angles on your own and check the results with your calculator. Good luck and have fun!

[a]See Module ***.

WWW

- What we did: SohCahToa is a memory aid for remembering the definitions of sine, cosine, and tangent ratios for angles that can fit in a right triangle.
- Why we did it: We found a simple way of finding the angles and side lengths of a right triangle using trigonometric functions.
- What's next: Next, learn the definitions of trigonmetric functions that are valid for any angles, not just the ones that can fit in a right triangle.